Drilling Engineering (2) (P.1)

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□ Drilling Objectives:

The goals of any drilling venture are safety, minimized cost and a usable completion.

Safety: Human Health

Human Welfare → Drilling Equipments

minimized cost: Drilling optimization

usable completion: Technology (state of art)

□ Drilling Optimization: involves using available resource to minimize cost subject to safety and well completion requirement.

to optimize drilling operation, we must do three things:

(1) Stablish Criteria for evaluating drilling performance.

(2) Identify the variables that effect this performance.

(3) Determine how to control these variables to our advantages.

Evaluating Drilling performance: ROP ↑ ⇒ cost ↓

$ROP = f\,(WOB, N, q, \text{Formation}, \ldots)$

□ Drilling Hydraulics:

The Drilling Hydraulics system have many effects on the well:

(1) Control subsurface pressure.

(2) provide a bouyant effect to drill string and casing.

(3) minimize hole erosion due to the mud's washing action during movement.

(4) Remove cutting from the well, clean the bit and remove cutting from below the bit.

(5) Increase the penetration rate. (WOB, N)

(6) Size surface equipment such as pump.

(7) Control "surge" pressure created by lowering pipe into the hole.

(8) minimize well-bore pressure reductions from "swabbing" when pulling pipe from the hole. (swab causes Kick and Blow-out)

(9) evaluate pressure increases in the well-bore.

(10) Maintain Control of the well during Kicks. (well-Killing ⇒ BOP)

▫ Drilling Problems:

(1) pipe stucking (Related to Loss Circulation)

(2) Wash-out (sudden hole in drill-string) ⇒ Collapse

(3) Well-bore Collapse

(4) Shale Problems (Swelling)

(5) Loss-Circulation (due to surge pressure)

(6) Lost of Bit (Bit Failure)

(7) Kick (due to swab pressure)

(8) Blow-out

(9) Deviated well Trajectory

▫ (1) Hydraustatic Pressure:

Common Form of Hydrostatic pressure equation are as follows:

$P_H = 0.052 \times \underbrace{MW}_{ppg} \times TVD$ (psi) $\left(0.052 = \frac{7.5\,(gal/cuft)}{144\,(in^2/ft^2)} \right)$

$P_H = \sum C\, \rho_i L_i$

C = constant

ρ_i = mud weight for the section of the interest.

L_i = TVD or Length of the section of Interest.

– Equivalent Mud weight (EMW)

$\underbrace{EMW}_{ppg} = \underbrace{P_{well}}_{psi} / (0.052 \times \underbrace{TVD}_{ft}) \rightarrow \{\text{static mud}\}$

$\underset{(lb/gal)}{EMW} = \frac{\text{total pressure} \times 19.23}{TVD}$

Drilling Engineering (2) (P.3)

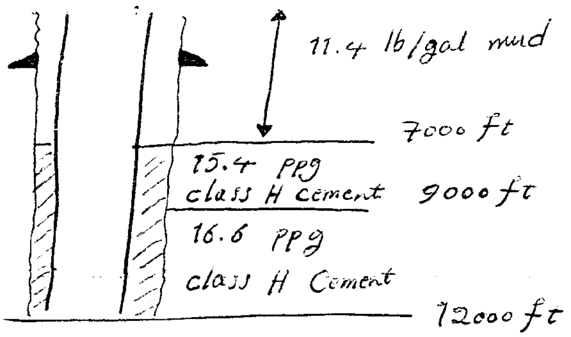
– Equivalent Circulating Density (PPG) → moving mud

$$ECD = \underbrace{MW}_{\text{static consideration}} + \underbrace{\frac{\Delta P_{annulus}}{0.052\ TVD}}_{\text{"motion of mud" consideration}}$$

$\Delta P(annulus) = ?$ (pressure Lost along the annulus)

✓ Note : ECD also is used for choosing mud to Kill the well.

□ Example: An Intermediate casing string will be cemented as shown. Calculate the hydrostatic pressure at 12000 ft. Convert the pressure at 12000 ft to an equivalent mud weight and determine if it will exceed the fracture gradient of 14.2 lb/gals.



Fracture Gradient = 14.2 ppg

$P_{at\ 12000'} = 0.052 \times 11.4 \times 7000 +$
$0.052 \times 15.4\ (2000) +$
$0.052 \times 16.6\ (3000) =$

$P(at\ 12000') = 8340$ psi

$$EMW = \frac{8340 \text{ psi}}{0.052 \times 12000} = 13.36 \text{ PPG}$$

13.36 < Fracture Gradient (14.2 ppg) ⇒ No problem.

□ Fracture pressure :

Formation Fracture Gradients define our upper well-bore pressure Limit.

Overbalanced Drilling ⇒ $P_{well} > P_{formation}$

$MW >$ Formation Fluid Pressure (Pore Pressure)

Balanced Drilling : $P_{well} = P_{formation}$

underbalanced Drilling : $P_{well} < P_{formation}$

Problems in: OVBD (Lost Circulation)

UBD (Kick $\Rightarrow$ Blow - Out)

BD (seldom is used in reality)

□ Mathews and Kelly (1967) on the landwork of Hubbert and Willis (1954) develops the following relationship for sedimentary rocks:

$$\gamma_{Frac} = \left[\frac{P_{pore} + K_i \, \sigma}{D} \right]$$

where:

γ_{Frac} = Fracture Gradient (psi/ft)

D = Depth (ft)

σ = Rock matrix pressure $\Rightarrow$ overburden pressure = Matrix Pressure + Formation Fluid Pressure

= σ + Pore Pressure

K_i = Matrix Stress Coefficient for the depth at which the value of σ would be the normal matrix stress (Dimensionless)

□ Mathews and Kelly procedure to determine the fracture gradient is as follows:

(1) Determine the pore pressure

(2) Assume a gradient of 1 psi/ft for the overburden pressure gradient

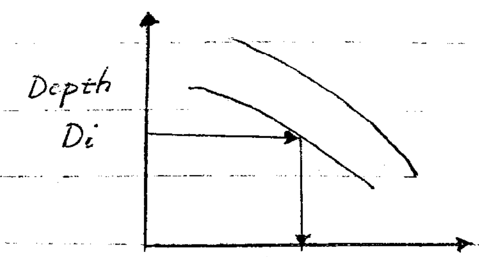
$= \sigma + P_{pore}$

(3) Determine the depth (D_i) at which σ would have normal value:

$\sigma = 0.535 \, D_i$ $\hookrightarrow$ (0.535 psi/ft)

(4) Using given figure to determine K_i and calculate γ_{Frac}.

$$\gamma_{Frac} = \frac{P_{pore} + K_i \, \sigma}{D}$$



K_i = Matrix Stress Coefficient

Gulf Coast Sands Layers, USA

Example: Determining Fracture Gradient

Using the Mathews and Kelly procedure, determine the fracture gradient Just below the casing seat for the following Louisiana Gulf Coast wells:

Casing at depth (seat depth) = 6650 ft, TVD

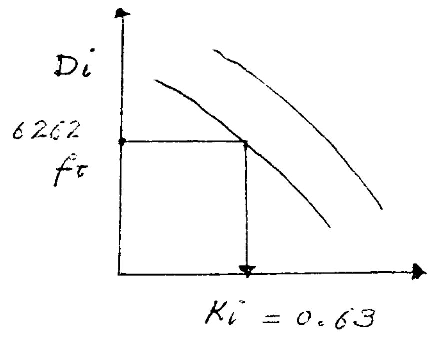
Formation pressure = 3300 psi

(1) $P_{pore} = 3300$ psi

(2) $\sigma = \quad - P$

$\sigma = (1\ psi/ft \times 6650\ ft) - 3300 = 3350$ psi

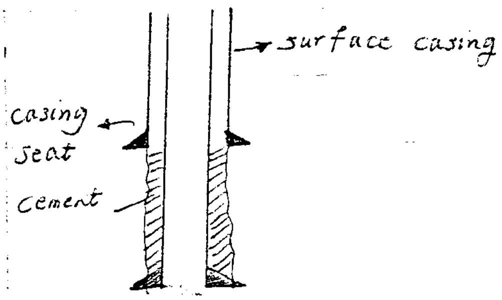
(3) $D_i = \dfrac{\sigma}{0.535} = \dfrac{3350}{0.535} = 6262\ ft \Rightarrow K_i = 0.63$



$$\gamma_{frac} = \frac{3300 + 0.63\,(3350)}{6650} = 0.814\ psi/ft$$

حساس‌ترین جای دهنه چاه Casing Seat است (Fracture احتمالی در این نقطه خواهد بروز.)

slow rate = 30 stroke/min (spm)



D_i : is the depth at which normal matrix pressure gradient occures.

slow rate برای عملیات کشتن چاه کاربرد دارد: ICP = SPRP + SIDPP ✓

Leak-off Test: (practical determination of fracture gradient)

In Field, we can estimate the "minimum" fracture gradient at each new casing point by performing Leak-off Test as follows:

(1) close the Blow-out preventor and apply pressure down the drill pipe in small increments using a Low-volume Pump.

(2) Continue pumping small volumes of mud until the pressure reaches a pre-test Limit. (The gauge is going to show a sudden decrease in this point)

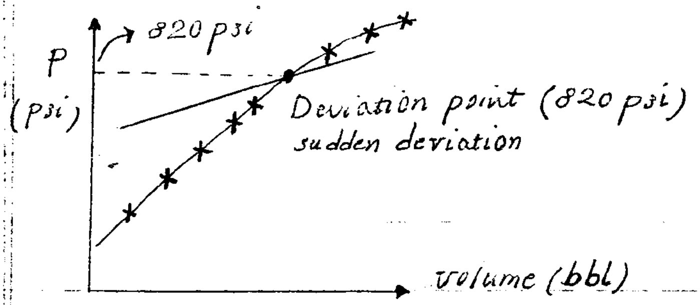
Note: This Limit is that pressure in which mud starts to invade the formation or alternatively fracture the formation, hence it is a "minimum" value.

□ Example: Leak-off Test

Calculate Fracture gradient

Determine the fracture gradient at a well's casing point (6750 ft, TVD), Given the following information in table below: (MW) = 12.5 ppg

volume pumped, bbl	pressure, psi
0.0	0
1.0	40
1.5	100
2.0	190
2.5	280
3.0	370
3.5	460
4.0	550
4.5	640
5.0	730
5.5	820 → Leak-off pressure
6.0	850
6.5	880



$$P_{frac} = 820\ psi + (0.052 \times 12.5 \times 6750)$$

$$P_{frac} = 5208\ psi$$

$$\gamma_{Frac} = \frac{5208\ psi}{6750\ ft} = 0.772\ psi/ft$$

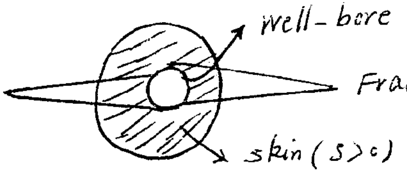
□ Therefore:

The Fracture Pressure = Applied pressure in Leak-off Test (From Fig. P vs Volume Fluid injected) + Hydrostatic well-bore pressure

ـ نکته : Leak-off Test تعیین می‌کند که در کدام عمق Casing Seat خواهد بود .

نکته: mini-fracture test همان Leak-off test است با این تفاوت که از مقدار "minimum" کمتر خواهد شد (mini-fracture test برای مطالعات Hydraulic Fracturing است). چون می خواهیم در دهانه چاه که آسیب دیده (s>0)، Fracture ایجاد کنیم.

Hydraulic Fracture Fluid: N_2 + Brine + Sands + CO_2 gas + Additives



- Bouyancy

Bouyant Force = Fluid Volume Displaced × Fluid Density

$W_{Fluid} <<< W_{Air}$

BF = Bouyancy Factor ⇒ $BF = 1 - \frac{MW}{\rho_{steel}}$, $\rho_{steel} = 65.5$ ppg or $7800\ Kg/m^3$

It is important to account for Bouyancy effects when calculating:

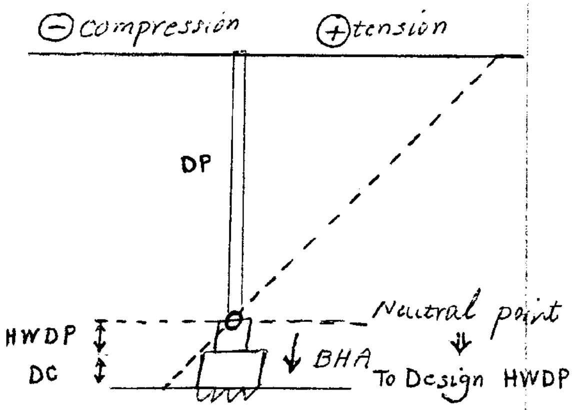
Hook Loads

Bit weight

Drill Collar requirements

Rig Capacity

Other weight-related parameters



- Example: Determine the maximum WOB provided by 450 ft of 7 3/4" O.D 144 lb/ft drill collar for both of the following mud weights. (Assume that all of the drill-string Compression is in drill collar)

(a) 9.5 lbm/gal (b) 16.0 lbm/gal

Solution : $W_{air} = 450 ft\ (144\ lbm/ft) = 64800\ lb$

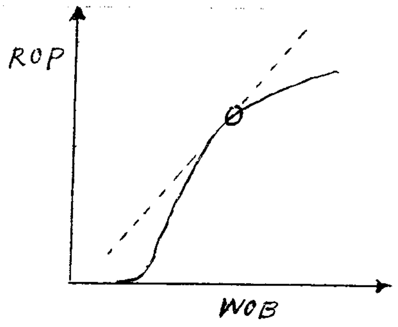
If: $MW = 9.5\ lbm/gal\ (PPG) \Rightarrow BF = 1 - \frac{9.5}{65.5} = 0.855$

(a) $W_{Fluid} = 64800 \times 0.855 = 55400\ lb$

(b) $MW = 16.0\ ppg$

IF $MW = 16.0\ ppg \Rightarrow BF = 1 - \frac{16}{65.5} = 0.756$

$W_{Fluid} = (64800)(0.756) = 49000\ lb$



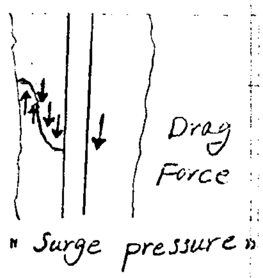
Note: Downhole motor (with MWD tools)

- **Surge and Swab pressures:**

Surge: when we run pipe in a well, it forces drilling mud up the annulus and out of the flow line, At the same time the mud immediately adjacent to the pipe is dragged downhole.

The resulting "piston effect" generates a surge pressure that is added to the hydrostatic pressure. Excessive surge pressures can increase the well-bore pressure to such a degree as to increase Lost Circulation. ⇒ pipe stucking.

Swab: When we pool pipe out of the well, mud flows down the annulus to fill the resulting void. This causes a "suction effect", generating a swab pressure that can Lower the differential pressure and possibly bring the formation fluid into the well-bore. ⇒ Kick ⇒ Blow-out



$\Delta P = P_{mud} - P_{formation}$

Calculating Surge and Swab pressure can be a complex undertaking, depending on the pipe configuration and hole geometry.

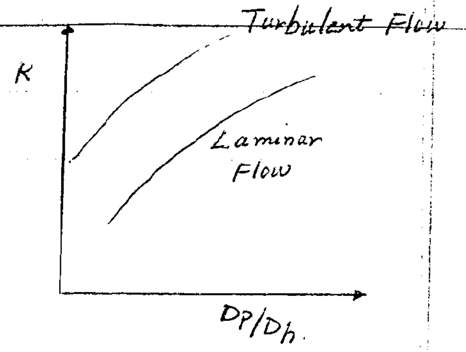
"Burkhardt" developed a relationship b/w well geometry and the effect of mud being dragged by the pipe which is refered to as the clinging constant. (K)

D_p = pipe Diameter

D_h = Hole Diameter

$V_{ms} = -K\,V_p$

V_p: pipe velocity



For a closed drill string: (e.g., inside BOP)

$$V_{mud} = -V_{pipe}\left[\frac{d_1^2}{d_2^2 - d_1^2}\right]$$

V_{pipe}: pipe velocity

d_2 = Hole Dimater

d_1 = pipe Diameter (O.D)

For an open-ended pipe:

$$V_{mud} = -V_{pipe}\left[\frac{4d_1^2(d_2-d_1)^2 - 3d_1^4}{4(d_2-d_1)^2(d_2^2-d_1^2) + 6d_1^4}\right]$$

Effective Annular Velocity: $\boxed{V_e = V_{mud} - K\,V_{pipe}}$

■ Example: Surge Effects

Calculating the surge pressure generated by running a string of 10 3/4" casing under the following condition and wether the total well-bore pressure exceeds the fracture gradient. Assume that the casing is effectively "closed" with a float shoe.

Casing point = 6400 ft

Fracture Gradient = 0.82 psi/ft

pipe velocity = −110 ft/min = −1.83 ft/s ("−" denotes downward velocity)

Hole Diameter = 14 3/4"

Mud: 15 PPG, PV = 37 cp, YP = 6 lbf/100 ft²

$$V_{mud} = -V_{pipe}\left(\frac{d_1^2}{d_2^2 - d_1^2}\right) = 1.83\left(\frac{10.75^2}{14.75^2 - 10.75^2}\right) = 2.073 \text{ ft/sec}$$

$d_1/d_2 = 10.75/14.175 = 0.75$ From Figure (Assume Laminar Flow): $K = 0.44$

$V_e = 2.07 - (0.44 \times 1.83) = 1.265$ ft/sec

use annular pressure Loss equation for Laminar Flow :

$$\Delta P_{ann} = \frac{PV \times L \times V}{1000\,(d_2 - d_1)^2} + \frac{YP \times L}{200\,(d_2 - d_1)} \quad \text{(Binghaum Fluid Assuming)}$$

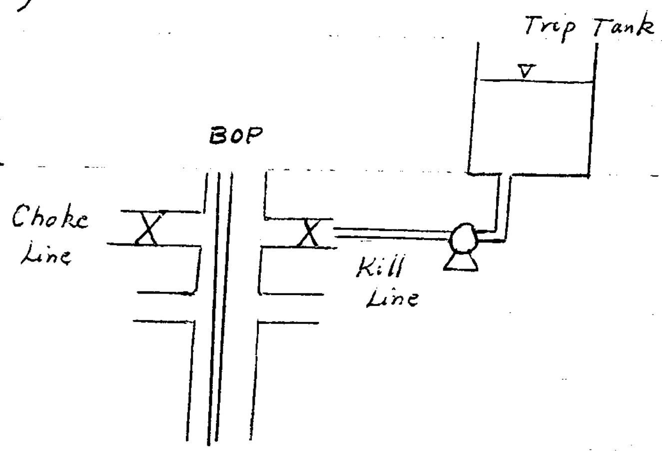
$$P_{surge} = \frac{37 \times 6400 \times 1.265}{1000\,(14.75 - 10.75)^2} + \frac{6 \times 6400}{200\,(14.75 - 10.75)} = 67 \text{ psi}$$

$$EMW = \frac{67}{0.052 \times 6400} + 15.0 = 15.2 \text{ lbm/gal}$$

$$EMW_{frac} = \frac{0.82}{0.052} = 15.77 \text{ lbm/gal}$$

$EMW < EMW_{frac}$ ⇒ ? We Can substitute the effective velocity into the friction pressure eqn. to compute the surge (swab) pressure.

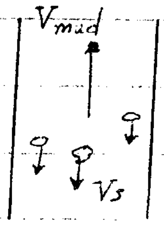
Note: Mud-Logging has been designed today for calculating surge or swab pressure. (showing by graphs)



نکته: تا زمانی که Trip Tank در موقع لوله‌بالا قادر به
تأمین حجم گل خالی شده درون فضای حلقوی
باشد swab pressure خواهیم داشت
اما بمحض افزایش سرعت لوله‌گذاری، افت
فشار ناگهانی داشته و Kick بوجود آمده و فوران
خواهیم داشت.

□ Hole Cleaning :

The mud's effectiveness at removing cuttings significantly affect drilling efficiency. Drilling Cutting vary in size and densities according to formation Lithology, Differential pressure, the cutting action of the bit and other factors. They are usually heavier than the drilling mud. and therefore tend to slip down through the annulus, back toward the bottom of the hole. A mud's ability to transport cutting, that is, its "Carrying capacity", is related to the difference b/w annular velocity and the slip velocity with which the cuttings fall.



$$\boxed{\text{Carrying Capacity} = V_{ann} - V_{slip}}$$

Note: Effect of Differential pressure on cutting size!

"Moore's Correlation" (1974) for estimating slip velocity: to use this method, we need to determine the average densities and diameters of drilled solids by visually inspecting. representative cuttings, or by doing a sieve or screen analysis.

We can use the following relationship to estimate the "slip velocity" of a particle suspended in a "Newtonian Fluid":

$$V_s = 1.89\sqrt{\frac{d_p(\rho_p - \rho_f)}{C_{drag} \times \rho_f}} \quad (*)$$

where: V_s: slip velocity (ft/sec)

d_p: particle diameter (inches)

ρ_p: particle density (PPg)

ρ_f: fluid density (PPg)

C_{drag}: Drag Coefficient

1.89 = Numerical value of Conversion constant, which has units of $\left(ft^2/(sec^2\text{-}in)\right)^{1/2}$ in SI unit 1.89 becomes 3.615

– Sieve Analysis: to determine cutting size distribution

to determine drag coefficient, we must first compute the particle's N_{Re}.

$$N_{Re,p} = \frac{928\,\rho_f \times V_s \times d_p}{\mu}$$

where:

μ: fluid Viscosity (cp) in SI unit (Kg/m^3, m/s, ...) 928 reduces to 1.

– روش محاسبه سعی و خطا است یعنی ابتدا N_{Re} را فرض کرده و محاسبات Trial and Error را انجام می‌دهیم که مشکل زا است

Moore presents the following modification of Eq. (*) for various ranges of the particle Reynolds number.

** For $N_{Re} < 1$, $C_{drag} = 40 / N_{Rep}$

$$V_s = \frac{82.87\, d_p^2 (\rho_p - \rho_f)}{\mu_a}$$ (μ_a: apparent (effective) viscosity)

In SI unit , 82.87 becomes 0.3267.

(**) For $N_{Re} > 2000$ (Turbulent Flow)

$C_{drag} = 1.5$

$$V_s = 1.54 \sqrt{\frac{d_p (\rho_p - \rho_f)}{\rho_f}}$$

In SI, 1.54 becomes 2.945 .

For Intermediate particle N_{Re} values , i.e , ($10 < N_{Re} < 100$)

$$V_s = \frac{2.9 \times d_p \times (\rho_p - \rho_f)^{0.667}}{(\rho_f)^{0.333} (\mu_a)^{0.333}}$$

~~In SI , 2.90 becomes 0.7053.~~

The term μ_a in these eqns. , is the apparent viscosity (cp) and defined as follows:

$$\mu_a = \left(\frac{K}{144}\right)\left(\frac{d_2 - d_1}{V_{ann}}\right)^{1-n}\left(\frac{2 + \frac{1}{n}}{0.0208}\right)^n$$

where: K is consistency factor

n: flow behaviour index $= 3.32 \, Log \left(\frac{\theta_{600}}{\theta_{300}}\right)$, $\boxed{K = \frac{\theta_{300}}{(511)^n}}$

In SI, μ_a (pa.s)

□ Example: Slip Velocity , Moore's Method

Calculate the slip velocity of drilling cuttings have an average diameter of 0.25 and a density of 2.65 gr/cc. (2.21 ppg)

given the following mud , hole data :

Hole size : 12.25"

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Drill pipe : 5"

Mud Weight (MW) = 10.2 ppg

V_{ann} = 60 ft/min = 1 ft/sec

n (from viscometer reading) = 0.8

K = 150 cp equivalent

Solution:

Determine μ_a :

$$\mu_a = \left(\frac{150}{144}\right)\left(\frac{12.25-5.0}{1.0}\right)\left(\frac{3.25}{0.0208}\right) \Rightarrow \mu_a = 88.1 \text{ cp}$$

2. Assume an $(N_{Re})_p$ range and then check the results, Let $(N_{Re})_p < 1$

$$v_s = \frac{82.87\,(0.25)^2(22.1-10.2)}{88.1} = 0.7 \text{ ft/sec}$$

$$N_{Re_p} = \frac{928 \times 10.2 \times 0.7 \times 0.25}{88.1} \gg 1$$

Assume : $10 < N_{Re_p} < 100$ (The most common oilfield condition)

$$V_s = \frac{2.9 \times 0.25 \times (22.1-10.2)^{0.667}}{(10.2)^{0.333}\ (88.1)^{0.333}} = 0.39 \text{ ft/sec}$$

N_{Re_p} = 10.5 (The assumption is valid)

3. Annular Velocity = 1.0 ft/sec

Net Upward Velocity of cuttings = (1.0 - 0.39) = 0.61 ft/sec

- وقتی Y.P در مدل Binghaum زیاد باشد افت فشار هم زیاد می‌شود چون Y.P تنش بیشتری را دریافت فشار مدل Binghaum از P.V خواهد داشت. Y.P بستگی به نوع Bentonite استفاده شده در گل را دارد.

$$\text{if } \left\{ Y.P = 0 \Rightarrow P.V \rightarrow \mu_e \right\}$$

(Newtonian Fluid)

- وقتی فرض سیال نیوتنی را برای گلی که در واقع بینگهام است می‌گیریم، افت فشار حاصله از هر دو نوع برابر خواهد بود. از طریق این معادل سازی میزان ویسکوزیته موثر (μ_e) را که تعریف کرده‌ایم بدست می‌آوریم. علت استفاده از μ_e این است که برای بدست آوردن N_{Re}، تعریف متوسط (ظاهری) ویسکوزیته را لازم داریم.

$$N_{Re} = \frac{700\, v D e}{\mu_e}$$

H.W : $N_{Re} = 3000$ (Use $N_{Re} = 3500$ instead of $N_{Re} = 3000$ as critical point) -
For pipe flow and annular flow (Bingham and power Law model)

- تفاوت در مقدار N_{Re_c} (بحرانی) به تفاوت در تعیین μ_p (P.V) در معادلات افت فشار منجر خواهد شد.

if $N_{Re_c} \uparrow \Rightarrow$ تنش P.V کاهش خواهد یافت (در جریان آرام)

The Critical N_{Re} depend upon: (1) Initial Turbulency of the flow
(2) Upstream edge
(3) Roughness

$$\text{if: } \begin{cases} \Delta P = f \dfrac{L}{D} \dfrac{V^2}{2g} \rho & \text{(Turbulent)} \\[2ex] \Delta P = \dfrac{32 L \mu_e V}{D^2} & \text{(Laminar)} \end{cases} \Rightarrow f = \frac{16}{Re}$$

- در عملیات حفاری زیری دائماً در حال تغییر است بنابراین مکانیک سیالات در مورد هیدرولیک حفاری بکار می‌رود. و برای محاسبهٔ زبری از معادلات تجربی استفاده می‌کنیم. (۱) معادله بلازیوس (Blausius eqn.) ⟵ $f = 0.057 Re^{-0.2}$
(۲) معادله پرانتل (Prandtl eqn.) ⟵ Trial and Error

- ویسکوزیته در جریان متلاطم متفاوت از جریان آرام است. بنابراین باید ویسکوزیته حالت متلاطم را استفاده کنیم که فرض بر سیال بینگهام است.

$$\text{Eddy Viscosity} = \frac{P.V}{3.2} = \mu_t \quad \text{(Turbulent)}$$

$$\left(\frac{PV}{3.2}\right) = \mu_t \quad \text{(Obtained by Trial and Error)}$$

P.V : plastic viscosity for Bingham Model Fluid

$M_T = \frac{P.V}{3.2}$ - در جریان متلاطم ، اگر مدل بینگهام نبود باید ویسکوزیته مدل را به P.V (برحسب P.V) تبدیل کنیم و :

Flow Through Nuzzle: Bernouli's eqn. ⇒ $\underline{\underline{V_n}}$ (is calculated) : nuzzle Velocity

Assume : (1) No friction Loss

(2) Neglect the potential head

(3) C_d : discharge Coefficient (Account for $\overset{\text{friction}}{\text{head}}$ Loss) $C_d \simeq 0.95$

(4) $\Delta P_{(nozzle)} \; \frac{\rho V_n^2}{2}$ (Dynamic Pressure)

Note: Drilling Optimization refers to Nuzzle Size.

$$\underset{\text{Facility}}{\Delta P_{surface}} + \Delta P_{DS} + \Delta P_{annular} + \Delta P_{Bit} = P_{pump}$$

$$\Downarrow$$

$$\Delta P_{Bit}$$

$$\Downarrow$$

$$V_n$$

$$\Downarrow$$

A_T (Total Nuzzle Area)

- وقتی گل به سطح زمین می رسد ، شرایط مطلوب تا آن است که فشار مانده گل ، نزدیک به فشار اتمسفر باشد .

Optimization of Bit Hydraulics

Minimum Cost Drilling

C = (Bit Costs) + (Trip Costs) + (Rotating or "on bottom" costs)

$$\left[\frac{C}{\Delta D}\right] = \frac{C_{bit} + C_{rig}\,(t+T)}{\Delta D} \quad \text{(cost per feet)}$$

where:

C: overall drilling Cost

ΔD: ft of interval Drilled (variable)

C_{bit}: Cost of bit (Fixed)

C_{rig}: hourly Rig Cost (Fixed)

t: Trip Time (hrs) (Fixed)

T: Rotating Time (hrs), (variable)

This equation is a basic tool in bit selection and in evaluating drilling performance under various sets of operating conditions. (WOB, N(RPM), q_{Pump}, MW, ...)
Various sets of operating conditions, We can use for both analyzing historical drilling data (i.e, from offset wells) and for monitoring the current bit run.

Fixed Parameters:

We can best evaluate cost per feet on the basis of single bit runs. This will provide us with a means of comparing individual bits and also allows us to make the following assumptions:

- Since the bit is already in the hole, C_{bit} is constant.
- Hourly Rig cost is unlikely to vary significantly during a bit run, we can therefore consider C_{rig} is constant.
- Trip Time (t) does not change during the bit run.

We can thus define bit cost, Rig cost and trip time as fixed parameters.

√(a) Bit Cost (C_{bit}): depending on a bit size, type and condition (i.e, New or used), may range from several hundred to tens of thousands of dollars.

We can group bit types into two basic categories:

✓Rolling Cutter, which include milled steel tooth and tangstan carbide insert bits.

✓Fixed Cutter, which includes steel cutter, natural diamond and polycrystalline diamond Compact (PDC) bits.

Selection of a parnclar bit type is based on offset well records (when available) or earlier bit runs on the current well. Major Considerations in bit selection includes the following:

- Formation hardness and Abrasiveness
- Mud Type (OBM, WBM, Air, Foam)
- Differential pressure (Amount of Overbalance)
- Directional or Horizontal Drilling Equipments
- Type of Rotating System (Rotary Table or Down-hole mud pump)
- Coring Equipment
- Hole Size

The Effect of bit selection on overall cost/ft depends not only on the bit's cost, but also on its performance. An inexpensive bit (or an expensive, high-performance bit) may or may not result in minimum cost per foot.

□ Example: Effect of bit Cost:

Compare the cost per feet of the bit runs shown in table below, given a Rig cost of \$ 200/hr and a trip time of 12 hrs.

Solution: $C/_{\Delta D} = ?$

Bit No.	Bit Cost	Rotating Time (T)	Footage (ΔD)
Bit 1	1200 \$	12.5 hrs	240 ft
Bit 2	4500 \$	24.4 hrs	504 ft
Bit 3	1200 \$	45.1 hrs	402 ft

Solution:

Bit 1: 25.42 \$/ft $\qquad \left[\frac{C}{\Delta D}\right]_1 = \dfrac{1200 + 200(12 + 12.5)}{240} = 25.42\ \$/ft$

Bit 2: \$ 23.57 /ft $\qquad \left[\frac{C}{\Delta D}\right]_2 = \dfrac{4500 + 200(12 + 24.4)}{504} = 23.57\ \$/ft$

Bit 3: \$ 25.96 /ft

Bit 2, which neither the cheapest nor the most expensive, never the less had the most economical run. Note that even though the bit costs vary significantly, differences in performance result in similar costs per feet.

(b) Rig Costs: reflects all of the operating expenses directly related to the drilling well. these include the equivalent hourly rate for drilling rigs and crew along with cost for:

- Rental Equipment (e.g, blow-out preventors, solid control equipment, drill string tools)
- Services (e.g, directional drilling or mud consultants)
- Mud Logging
- Drilling mud material and services
- Transport of drilling equipment and materials
- Allocated supervision and administration

Examples of items that would not be included in hourly drilling costs are:

- Site preparation and clean up (Clean-up : Remediation برگرداندن طبیعت به حالت اولیه)
- Casing, tubing and completion equipment
- Well-head Equipments
- formation Evaluation, including Logging and testing.
- materials and services for running casing, cementing, perforating and running production equipments
- Stimulation or sand control
- Supervision and administration, unless specifically allocated to the job
- Most well problems

Rig cost vary considerably according to supply and demand, Location environment, well requirements, rig type, standard equipment and contract provisions.

We should note that the cheapest hourly cost may not necessarily result in minimum cost per foot. These involved in Rig selection must also consider the efficiency of the contractors personnel and equipment.

A highly competent crew and a well-maintained rig may satisfy extra expense.

✓(C) The Trip Time (t) required to run and pull a bit depends on such factors as:

- Well depth
- Hole Size
- Required mud trip margin (Surge and Swab prevention)
- Bottom-hole assembly Configuration (Down-hole mud pumps or Geostrings)
- Presence of Hole problems
- Hoisting Capacity
- Rig and Crew Efficiency

A common "rule of thumb" for estimating trip time and one that is reasonably accurate over the life of a well, is to assume one hour of trip time to run or pull 1000 ft of pipe. (1 hr / 1000 ft)

✓Table below can be used to estimate trip times for various hole sizes:

	Hole Size, inches (cm)		
Depth (ft)	< 8.75	8.75 - 9.875	> 9.875
2000	1.5	3.0	4.5
4000	2.5	4.2	5.75
6000	3.5	5.4	7.0

Depth (ft)	Hole size, inches (cm) < 8.75	8.75 – 9.875	> 9.875
8000	4.7	6.5	8
10000	5.8	7.25	9
12000	7.0	8.25	10.25
14000	8.25	9.25	11.25
16000	9.75	10.25	12.50
18000	11.0	11.25	13.75
20000	11.8	12.25	15.0

Note: Top drive system reduces trip time. (The main top-drive advantages with respect to rotary table technology)

A-2: Variable Parameters

✓ Rotating Hours (T)

✓ Drilled Depth (ΔD)

Both of these two depend on a wide range of factors, some of which may be changed during the bit run.

The average penetration rate for a bit run is $\left(\frac{\Delta D}{T}\right)$, while the instantaneous penetration rate at any given time is defined as $\left(\frac{dD}{dT}\right)$.

We might expect that the Longest the bit runs and/or highest penetration rates result in minimum cost per foot. this is true in many cases. But as the following example shows, intitution can sometimes be misleading.

Example: Effect of footage and rotating hours: determines cost/ft of bit runs shown in table below, given a rig cost of \$275/hours and a trip time of 9.5 hrs, each bit costs \$4300.

Bit NO.	Rotating Time (h)	Footage (D)	Average penetration Rate
Bit 1	18.7	309 ft	16.5 ft/hr
Bit 2	24.5	374 ft	15.3 ft/hr
Bit 3	30.0	399 ft	13.3 ft/hr

Solution:

Bit (1) : $\left[\frac{C}{\Delta D}\right] = 39.01$ \$/ft $= \frac{4300 + 275\ (9.5 + 18.7)}{309}$

Bit (2) : $\left[\frac{C}{\Delta D}\right] = 36.50$ \$/ft

Bit (3) : $\left[\frac{C}{\Delta D}\right] = 38.00$ \$/ft

bit (2) has neither the Longest run the most footage nor the highest average penetration rate, it was nevertheles the most economical.

■ A.3 : Cost allocations

The overall cost/ft of a bit run is equal to the sum of the its fixed and variable costs, in terms of cost per foot:

$$\left[\frac{C}{\Delta D}\right] = \left[\frac{C_{fixed}}{\Delta D}\right] + \left[\frac{C_{variable}}{\Delta D}\right]$$

where: $\left[\frac{C_{fixed}}{\Delta D}\right] = \frac{C_{bit} + (C_{rig} \times t)}{\Delta D}$ (1)

$\left[\frac{C_{variable}}{\Delta D}\right] = \frac{C_{rig} \times T}{\Delta D}$ (2) $\left(\frac{cost}{ft} \propto \frac{1}{ROP}\right)$

✓ This eqn(1) indicates that where C_{bit}, C_{rig}, t are constant, drilled depth is the governing parameter in determining fixed costs.

✓ While the other (2) shows an inverse relationship b/w variable cost and rate of penetration.

$\frac{C_{variable}}{\Delta D} \propto 1 / \frac{\Delta D}{T}$ Penetration Rate

$C_{variable} \uparrow$ when $\frac{\Delta D}{T} \downarrow$ Low R

$C_{variable} \downarrow$ when $\frac{\Delta D}{T} \uparrow$ High R

✓ The relative contributions of fixed and variable cost to overall drilling cost can change significantly during a bit run.

When a bit starts to drill, most of the operating expense is attributable to bit cost and trip time. As the bit continues to drill, [most of the operating expense is], the fixed costs begins to decrease. At the same time the variable costs increases until they eventually overtake the fixed costs. the net effect of of these changing contributions in the overall cost per foot decreases from a high initial value to a minimum, and then begins to increase as the bit dulls.

□ Example: Fixed and variable Costs:

Given the information below, calculate the fixed and variable cost for the following bit run. Determine the time at which the overall cost per foot reached a minimum.

Rig Cost : $ 200/hr

Trip Time : 8 hr

Average bit weight : 60000 lbf

mud weight : 10.0 lbm/gal

Bit Costs : $ 5200

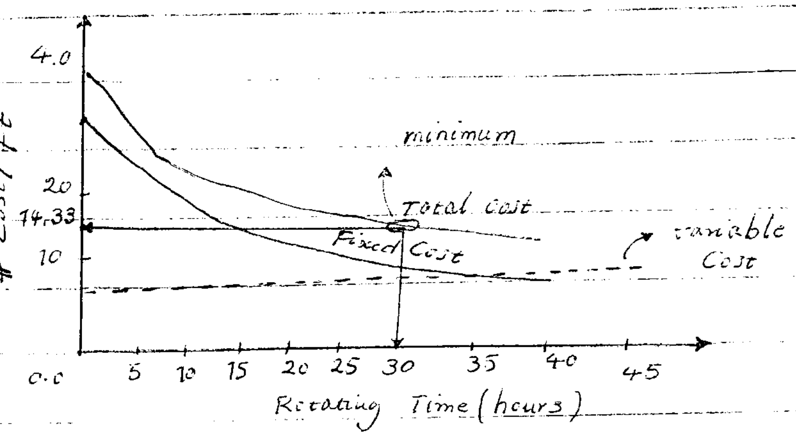
Bit pulled at 45 hr.

Average Rotating Speed : 90 RPM

	(1) ΔD (ft)	(2) T (hr)	dD/dT (ft/hr)	(3) $\frac{C_{fixed}}{\Delta D}$ (\$/ft)	(4) $\frac{C_{variable}}{\Delta D}$ (\$/ft)	(5) $C/\Delta D$ (\$/ft)
	40	1	40	170.0	5	175
	200	5	40	34.0	5	39
	392	10	38.4	17.35	5.102	22.45
	550	15	31.6	12.36	5.46	17.818
	644	20	22.8	10.55	6.21	16.77
	806	25	22.4	8.44	6.203	14.64
□	893	30	17.4	7.61	6.71	14.333
△	937	35	8.8	7.26	7.47	14.73
○	971	40	6.8	7.00	8.239	15.24
	998	45	5.4	6.81	9.02	15.83

□ Minimum Overall Cost (Answer), △ : Variable Cost Overtake the Fixed Costs.

○ Overall Cost Tend to Increase after a Minimum (i.e, Bit Dulls)



A-4: Practical Applications:

In the preceding example we determined the min. cost/ft for a specific set of operating conditions (BW: 60000 lbf, speed = 90 rpm, MW = 10.0 ppg). We did not address the issue of how these conditions might have affected the bit run and our subsequent calculations. Would a different set of Conditions have resulted in better drilling performance and Lower cost per foot. What are these?

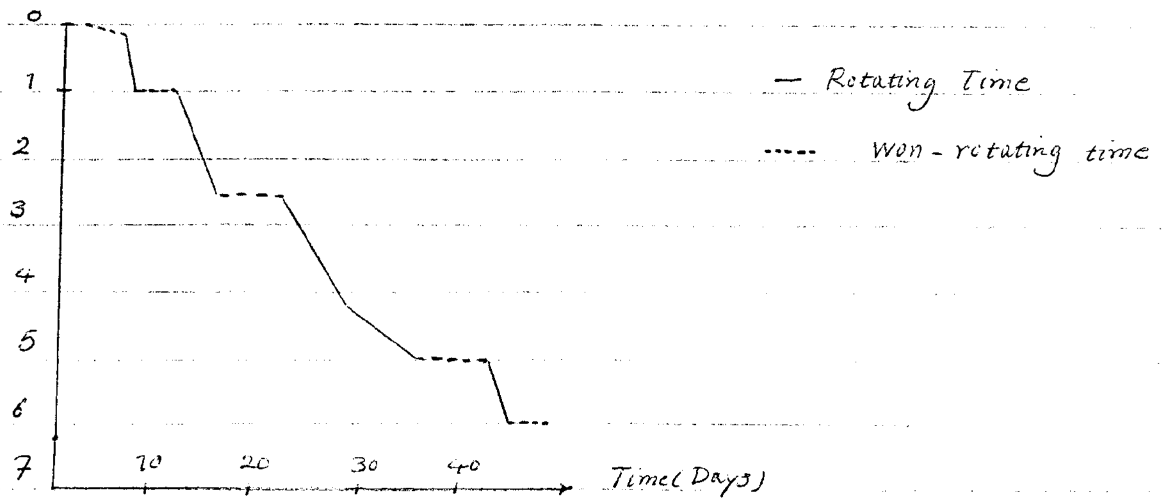
There is an effective balance b/w drilling parameters result in minimum cost drilling. the drilling engineer's job is to establish this balance by:

- ✓ Identifying variables that affect these parameters
- ✓ Determining what conditions of these variables most favourably influence cost per foot.
- ✓ Since bit cost, Rig cost and trip time are constants for a single bit run, our primary concerns are rotating hrs (i.e, bit life), drilled footage and instantaneous penetration rate (dD/dt).

Drilling Optimization:

Successful Drilling Engineers know not only how to deal with well problems but also how to optimize normal routine operations (RPM(N), WOB, mud). They are always monitoring, comparing and analyzing performance in their efforts to provide the best well Completions of the Lowest cost.

✓Our first task in drilling optimization, of course, is to stay out of trouble. Hole problem aside, it is drilling to target depth, or making hole, that typically takes up the greatest amount of a rig's time.



Therefore a primary concern in optimizing operation, we should emphasize, however, that the optimization task encompasses all aspects of a well program, from pre-spud planning to final completion.

A: Minimum Cost Drilling

Fixed parameters

Variable Parameters

Cost Allocations

Practical Applications

B: Factors affecting the ROP:

Formation properties, mud and bit type

Hydraulics

Weight on Bit

Rotary Speed

C: Penetration Equations: Bourgone and Young's Model describing bit run

D : Drilling Tests

Fire spot test

Drill-off test

E : Constrains on WOB and Rotary Speed

F : Estimating Bit Life

Bearing Wear

G: Implementation Procedure

(B) penetration Rate

There are many variables that affect "how fast" and for "how Long" we can drill a given interval.

✓They include :

formation properties

Mud Properties

Hydraulics

Bit Type

Weight On Bit

Rotary speed

Bit tooth wear

The relationships b/w each of these variables and drilling performance may involve many unknowns, making it difficult to develop a comprehensive drilling model.

To get around this difficulties, researches typically establish conditions when they can study the effects of one variables while holding others constant.

Correlations are actually most empirical, and are based on Laboratory and field data for specific areas or rock types.

B-1) Formation Properties, mud properties and Bit Type :

The most important formation property with respect to drilling performance :

√ Compressive Strength and elastic Limit

√ Porosity

√ permeability

√ A highly porous, permeable formation with Low compressive strength generally exhibits higher penetration rates than a high-strength "tight" formation.

formation depth ↑ ⇒ ϕ ↓ ⇒ higher Compressive Strength, Consequently Lower penetration rates.

Mineralogical Characteristics are also important in determining drilling performance, primary examples of these characteristics are abrassiveness and Hydration.

√ Mud Properties also affect the peneteration rate, in a normal-pressure environment, differential pressure b/w the well-bore and the formation increases with increasing mud weight, inhibiting cuttings remove and causing penetration rates to decrease.

peneteration rates also tend to decrease with increasing viscosity and solid contents, while they usually increase with higher filteration rates.

- Higher (ΔP) b/w well-bore and formation ⇒ PiPe-stucking

- در UBD چیزی که باعث mud invasion می‌شود اختلاف فشار نیست بلکه capillary Effects می‌باشد.

- در مواقعی که Gas Kick می‌باشد باید حفاری متوقف شود ولی موارد دیگر به حجم در حال حفاری نیز بستگی دارد.

- تعیین نوع Fluid Kick : خواندن فشارِ Casing و داشتن حجم تعیین نوع سیال را امکان پذیر خواهد کرد.

Bit Type :

The Bit type (size) used for a given interval depends mainly on completion requirements for pipe size while the bit type depends on formation.

✓ Formation properties are critical in determining drilling performance. at the same time, they are beyond our control. mud properties and bit type though they are controllable, do not change significantly during a normal bit run, we can however control hydraulic, bit weight and rotary speed. (pump, WOB, RPM).

B-2) Hydraulic

Drilling performance depends on how well we remove cuttings from the bottom of hole. If hole cleaning is inadequate, the bit flunder, that is, its penetration rate decrease because it is regrinding unremoved cuttings or becoming burried in the formation. fortunately, we can exercise a great deal of control over hole cleaning, simply by varying a bit jet nuzzle diameters. our objective is to deliver an optimum amount of hydraulic energy through these nuzzles. In addition to remove cuttings, this energy works to cool the bit.

✓ Hydraulic Energy is related to Loss: $\Delta P_{bit} = \Delta P_{Pump} - \Delta P_f$

ΔP_f represents the sum of the pressure Losses in the surface equipment, drill pipe, bottom assembly and annulus.

✓ The most commonly optimized bit hydraulics in terms of: Hydraulic Horse Power, impact force (IF) or nuzzle velocity.

$$HHP = \frac{(\Delta P_{bit})\, q}{1714}$$ where q: circulation rate (gpm), ΔP_{bit} (psi)

$$IF = 0.01823\, C_d \times q \sqrt{\Delta P_{bit} \times MW}$$ where: IF: Impact Force (lb_f)

MW: Mud Weight (lbm/gal)

C_d: Nuzzle Discharge Coefficient (0.95)

$$V_n = \overbrace{0.32086}^{(144/7.5\times 60)} \left(\frac{q}{A_T}\right)$$ where: V_n: nozzle Velocity (ft/sec)

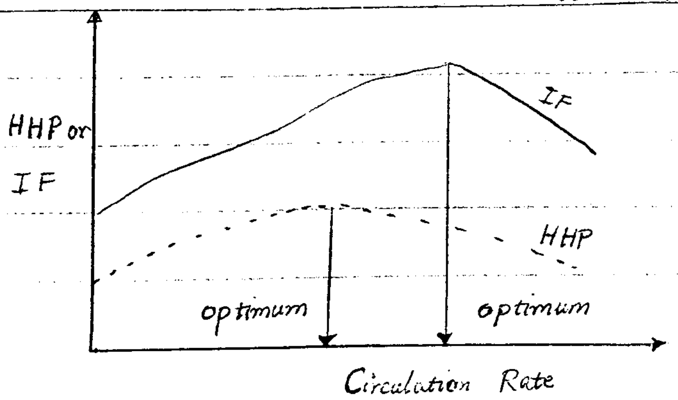
q: flow rate through nuzzle (gal/min)

A_T: Total Nuzzle Area (in^2)

$$\boxed{\Delta P_{bit} = \frac{q^2 \times MW}{12031\, C_d^2\, A_T^2}}$$

✓ Because of the higher friction pressure that accompanies increased circulation

Con'd → Rates, hydraulic Horse Power or impact force is limited by the pressure rating of the mud Pumps. These are optimum circulation rates for which HHP, & IF are maximum (See Figure).



At higher rate, friction Loss become excessive, we can show mathematically that maximum HHP occures when:

$$\boxed{\Delta P_f = \frac{P_{pump}}{n+1}}$$

and that maximum bit impact force occures when:

$$\boxed{\Delta P_f = \frac{2\,P_{pump}}{n+2}}$$

when "n" is flow exponent determined from a Logarithmic plot of "P" vs. flowrate at two points:

$$n = \frac{\text{Log}\,(P_1/P_2)}{\text{Log}\,(q_1/q_2)}$$

□ Example: Optimizing HHP and Impact Force (IF)

Determine the optimum jet size, based on hydraulic horse power or impact force criteria, for the following well:

Pump Rate # 1 : 420 gpm at 3000 psia

Pump Rate # 2 : 275 gpm at 1300 psia

Pump HP = 1250 HP

MW = 13 lbm/gal

Nozzle Size = 17/32 in diameter (3 nozzles)

Minimum Annular Velocity = 70 ft/min (1.16 ft/sec)

Minimum Pump Rate = 175 gpm (For Optimizing Nozzle Velocity)

Drilling Engineering (2) (P. 30)

Hole Size : 8.5"

Drill pipe : 4.5"

Drill collar : 7"

Solution: Maximum q:

$$HHP = \frac{Pq}{1714} = \frac{3000 \times q}{1714} \Rightarrow q = 714 \text{ gpm}$$

Minimum q based on minimum Annular Velocity:

$$V = \frac{q}{2.448(d_2^2 - d_1^2)}$$

$$1.167 = \frac{q}{2.448(8.5^2 - 4.5^2)} \Rightarrow q = 148 \text{ gpm}$$

Friction Pressure Losses:

Rate 1 :

$$\Delta P_{f_1} = -\frac{(MW)\, q^2}{12031\,(C_d^2)(A_t^2)} + P_{pump}$$

$$\Delta P_{f_1} = 3000 - \frac{(13)(420^2)}{(12031)(0.95)^2\left(\left(\frac{3\pi}{4}\right)\left(\frac{17}{32}\right)^2\right)^2} \Rightarrow \Delta P_{f_1} = 2523 \text{ psi} \quad \{\text{Rate } 1\}$$

Rate 2 :

$$\Delta P_{f_2} = 1300 - \frac{(13)(275)^2}{(12031)(0.95^2)\left(\frac{3\pi}{4}\left(17/32\right)^2\right)^2}$$

$$\Delta P_{f_2} = 1095 \text{ psi}$$

Flow exponent (n):

$$n = \frac{Log(P_1/P_2)}{Log(q_1/q_2)} = \frac{Log(3000/1300)}{Log(420/275)} = 1.97$$

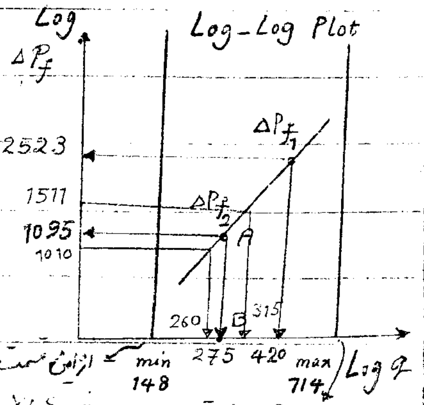
optimum pressure Losses, flowrate and nozzle size:

HHP: $\Delta P_f = \frac{3000}{1+1.97} = 1010$

optimum flowrate: (see fig.) = 260 gpm

$$1990 = \frac{(MW)(q^2)}{12031 \times C_d^2 \times A_T^2}, \quad \Delta P_{bit} = 3000 - 1010 = 1990 \text{ psi}$$

$$A_T = \left(\frac{13 \times 260^2}{12031 \times 0.95^2 \times 1990}\right)^{1/2} = 0.2017 \Rightarrow$$



ازاین قسمت به بعد خرده ها بالا نمی آید

از این بیشتر پمپ قادر نیست

Nozzle Area: $\frac{0.2017}{3} = 0.06723 \text{ in}^2$

$0.06723 = \left(\frac{\pi}{4}\right)\left(\frac{d}{32}\right)^2 \Rightarrow d = 9.36$

Use one $(9/32)$ and two $(10/32)$ nozzles.

Impact Force:

Optimum $\Delta P_f = \frac{2 \times 3000}{1.97+2} = 1511$ psi

Optimum Flowrate (Fig. 6-4): 315 gal/min

$\Delta P_{bit} = 3000 - 1511 = 1489$ psi

$A_T = \left(\frac{13 \times 315^2}{1201 \times 0.95^2 \times 1489}\right)^{1/2} = 0.2825 \text{ in}^2 \Rightarrow \frac{0.2825}{3} = \left(\frac{\pi}{4}\right)\left(\frac{d}{32}\right)^2 \Rightarrow d = 11.08$

Use $3(11/32)$ nds nozzle.

Important:

On some wells, we may not be apply HHP or impact force criteria because of limited pump capacity, high friction pressures or annular velocity restrictions. In these cases, nozzle velocity becomes our optimization criterion.

Maximum jet velocity occures when ΔP_{bit} is a maximum value for some established minimum flow rate (generally the lowest flowrate needed to overcome slip velocity).

Example: Optimizing Nozzle Velocity

Using the well data from previous example, Determine the optimum nozzle sizes based on maximum jet velocity, to lesson the risk of plugging the nozzles, do not use jet sizes less than 8/32 nds.

Solution: Minimum Pump Rate = 175 gal/min

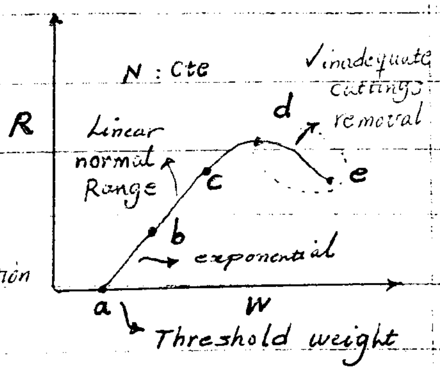
From Fig 6.4, $\Delta P_f = 470$ psi at 175 gal/min $\Rightarrow \Delta P_{bit} = 3000 - 470 = 2530$ psi

Jet Sizes: $A_T = \sqrt{\frac{13 \times 175^2}{12031 \times 0.95^2 \times 2530}} = 0.120 \text{ in}^2 \Rightarrow 0.12 = \frac{3\pi}{4}\left(\frac{d}{32}\right)^2 \Rightarrow d = 7$

Optimum nozzle size: three 7/32 nds, which is below the 8/32 nds limit.

(B-3) Weight on Bit:

Where all other factors are constant, penetration rate tends to increase with increasing WOB. Figure below illustrate this trend:



From this illustration we may observe the following:

There is a minimum or threshold bit weight (point a) below which the bit does not penetrate the formation.

Once the driller exceeds this threshold weight, penetration rate increases rapidly. (a to b) (exponential)

Within the normal range of weight on bit used in drilling operation (b to c) $\frac{dD}{dT}$ increases Linearly with W. (Linear)

Beyond this normal operating range, increasing bit weight result only in slight penetration increases (c to d). At extremely high wt values (or if bottom hole hydraulic are poor i.e poor cleaning) penetration rate decreases.

Actually decrease because of inadequate cuttings removal, or because the cutting element are being buried in formation, we refer to this condition as "bit floundering". (Causing Wash-out due to torsion)

Over the "normal" operating range of bit weights, we can express relationship b/w bit weight and instantaneous penetration rate as follows:

$$\boxed{\frac{dD}{dT} \propto (W - W_o)^{a_s}}$$

Where:

W_o: threshold bit weight, for consolidated or hard rocks $W_o > 0$, for soft rocks W_o may be equal to zero or for formations that can be drilled by jetting or washing the hole, Less than zero.

a_s: Bit weight exponent, which is constant for "a given" set of operating conditions.

Note: By hydraulic Jet in soft formations we will make horizontal wells.

Washout may occur due to: (1) Bit Flaunder ⇒ Causing Torsion ⇒ Causing Wash-out

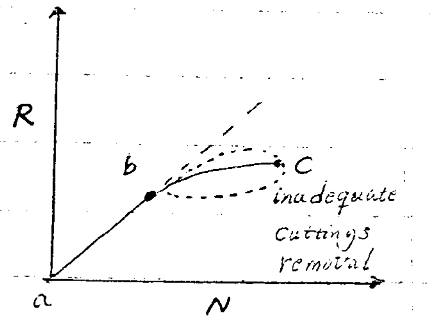
(۲) نفوذ مواد شیمیایی از درون سازند به گل حفاری و اثر این مواد شیمیایی بر رشته حفاری در نقاط ضعیف نیز به مسئله Wash-out می‌انجامد.

(۳) لرزش لوله حفاری و عدم وجود لرزه‌گیر (Vibration damp ner) نیز عامل دیگری در رخ دادن پدیده Wash-out می‌باشد.

(۴) عدم استحکام مواد اولیه لوله حفاری نیز عامل دیگری است.

(B-4) Rotary Speed

As a rule, penetration rate increases non-linearly with increasing rotary speed. Figure below illustrates this fact:



Note that as "N" increases past a certain point, the penetration rate does not increase as quickly. As the case for extremely high bit weight, this is a consequence of inadequate cutting removals of high rotary speed.

We may express the relationship b/w instantaneous penetration rate and rotary speed as follows: $\frac{dD}{dT} \propto N^{a_6}$

Where:

"a_6" is rotary speed exponent.

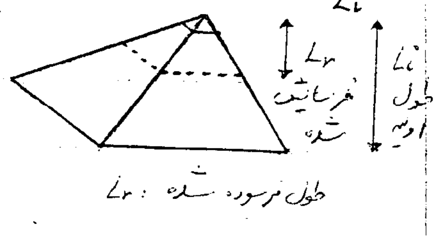
(B-5) Bit Tooth Wear

As a bit run progresses, tooth wear causes a gradual decrease in ROP. Bit manufacturers can reduce the effect of this wear, to some degree by selectively hard-facing the bit teeth, which result in a self-sharpening action. Hardfacing, However **does not** compensate for the reduction in tooth Length caused by abrasion and chipping. In many cases, of course, cutting elements may be broken or lost completely.

The decrease in ROP with increasing tooth wear is "non-Linear" and reason for this behaviour is that each tooth, as it wears down, presents a Larger cross-section area to the formation (Fig.)

A simple expression of this relationship is: $\frac{dD}{dT} \propto e^{-(H)a_7}$

$H = \frac{L_r}{L_i}$



Where:

H is the fraction of tooth height that has been worn away, expressed as a decimal (e.g., for an IADC tooth wear code of **T-5**, $H = 0.625$), H is reported by IADC standards for bit type.
«مدرسه»

C: Penetration Rate Equations:

Researchers have made various attempts to combine drilling variables into a single optimization.

□ Bingham's Equation: introduced in our discussion of the dc exponent is one commonly used example:

$$\boxed{\frac{dD}{dT} = 60 \times a \left(\frac{W}{d_B}\right)^{b} \times N}$$

Where:

$\frac{dD}{dT}$: Penetration Rate (ft/hr)

a: formation drillability constant (dimensionless)

W: WOB (lbf = 1000)

d_B: Bit Diameter (in)

b: Bit Weight exponent (dimensionless)

N: Rotary Speed, RPM

This empirical equation is commonly used in field calculations, and has been adapted as a tool for predicting pore pressure. It assumes a threshold bit wt of zero, a value of one (1) for the rotary speed exponent and perfect bottom hole cleaning. ✓ زمانی مقدار Threshold Bit Weight برابر صفر است که یعنی اعمال وزن روی مته حفاری را شروع کنیم

It accounts for formation and other effects by assigning "a" and "b" based on Local drilling conditions (by choosing a, b other parameters which effect on ROP are constant)

This equation is used as a tool for predicting pore pressure.

(C-1) Bourgoyne and Young's model:

Bourgoyne, At. Jr, E.S., Young, Jr (1974)

"A multiple regression Approach" to optimal Drilling and abnormal pressure detection, society of petroleum engineers Journal (August). Richardson, tx; society of petroleum engineers.

They developed the following drilling eqn. for rolling cutter bit, which incorporate all of the drilling parameters identified in this discussion: $\frac{dD}{dT} = \exp\left(a_1 + \sum_{i=1}^{8} x_i a_i\right)$

(x_2 to x_8)

$$\frac{dD}{dT} = \left(e^{a_1}\right)\left(e^{x_2 a_2}\right)\left(e^{x_3 a_3}\right)\left(e^{x_4 a_4}\right)\left(e^{x_5 a_5}\right)\left(e^{x_6 a_6}\right)\left(e^{x_7 a_7}\right)\left(e^{x_8 a_8}\right)$$

Field Units : a_1 through a_8 constant:

$$\left.\begin{aligned} x_2 &= 10000 - TVD \\ x_3 &= (TVD)^{0.69}\,(EMW_{Pore} - 9) \end{aligned}\right\}$$ Effect of formation Compaction

$x_4 = TVD \times (EMW_{Pore} - ECD)$ Effect of differential pressure

$x_5 = \ln\left(\dfrac{(W/d_B) - (W_0/d_B)}{4 - (W_0/d_B)}\right)$ Effect of bit weight and bit diameter

$x_6 = \ln(N/100)$ Effect of Rotary Speed

$x_7 = -H$ Effect of Bit Wear

$x_8 = \ln(IF/1000)$ Effect of Bit Hydraulic

The terms $a_1 - a_8$ are constant reflecting Local drilling conditions, which the terms x_2 through x_8 are functions denied from published Correlations.

Where: $a_8 x_8 \Rightarrow a_8$ is hydraulic exponent.

$a_2 x_2$, $x_3 a_3$: effect of formation compaction

a_2, a_3 : are formation exponent and depends on ϕ

$a_4 x_4$: $\Rightarrow$ a_4 is mud exponent

$a_5 x_5$: $\Rightarrow$ a_5 is bit weight exponent

$a_6 x_6 \Rightarrow$ a_6 is rotary exponent number

$a_7 x_7 \Rightarrow$ a_7 is bit wear exponent number

TVD (ft) : True Vertical Depth

EMW_{Pore} (ppg EMW) : Pore Pressure fluid gradient

ECD (ppg) : Equivalent Circulating Density

a_5 (bit weight exponent)

W (1000-lbf) : weight on bit , W_0 : Threshold of Bit weight (1000-lbf)

d_B (in) : Bit Diameter

a_6 : Rotary exponent number

N (RPM) : Rotary Speed

H : fraction of teeth that has been worn away, expressed as a decimal (e.g for IADC grade of T-5, H = 0.625)

IF (lbf) : Hydraulic Impact Force

Bourgoyne and Young developed this equation for "normalized" drilling conditions in a "given" formation, namely:

✓ Normal Compaction of 10000 depth

✓ Pore Pressure Gradient of 9.0 lbm/gal equivalent mud weight

✓ Drilling with a new bit at a differential pressure of zero

✓ A bit weight of 4000 lbf per inch of bit diameter and RPM of 100.

✓ Hydraulic Impact Force of 1000 lbf of bit nozzles.

✓ The term e^{a_1} : is a drillability constant that describes the effect of formation strength and bit type on the ROP, as well as effect of drilling variables that have not been mathematically modeled. It has the units of ft/hr and is numerically equal to the ROP that would be attained under the defined "normalized conditions" above under this conditions. $\boxed{\frac{dD}{dT} = e^{a_1}}$

✓ The term $e^{a_2 x_2}$ through $e^{a_8 x_8}$ are multiplers:

That is, if a condition's net effect is to decrease the "normalized" penetration rate, then the corresponding term $e^{x_2 a_2}$ is less than one, on the other hand, it works to increase ROP over the normalized value, $e^{x_2 a_2}$ is greater than one.

Example: Estimating Formation Drillability

Use Bourgoyne and Young's drilling model to estimate the apparent drillability (e^{a_1}) for a shale formations, under the following conditions:

Drilling Engineering (2) (P. 37)

TVD = 10850 ft

Pore Pressure = 5200 psi = 9.2 ppg EMW

ECD = 10.0 ppg : Equivalent Circulating Density

Bit size: 7 7/8 in , New bit: H=0

Bit Weight: 42000 lb_f (assume threshold wt of bit = 0)

Rotary speed = 120 RPM

ROP = 16.3 ft/hr

Calculated Hydraulic IF = 1200 lb_f

Based on historic well data, the estimated values of a_2 through a_8 are:

$a_2 = 0.0002$ $a_3 = 0.0003$ $a_4 = 0.0003$ $a_5 = 1.0$ $a_6 = 0.6$ $a_7 = 0.7$ $a_8 = 0.5$

Solution:

$$e^{a_2 x_2} = e^{(0.0002(10000 - 10850))} = 0.8437 \quad , e^{x_3 a_3} = 1.0377$$

$$e^{a_4 x_4} = 0.7707 = e^{(0.0003 \times 10850 (9.2 - 10))}$$

$$e^{a_5 x_5} = 1.333 = e^{(1 \times \ln[(W/d_B) - (W_t/d_B)])}$$

$$e^{a_6 x_6} = 1.136 = e^{(0.6 \ln(120/100))} = 1.116$$

$$e^{a_7 x_7} = 1.0 = e^{-0.7H} = e^0$$

$$e^{a_8 x_8} = 1.095 = e^{0.5 \ln(IF/1000)} = e^{0.5 \ln(1200/1000)}$$

$$\frac{dD}{dT} = e^{a_1}(0.8437)(1.037)(0.7707)(1.333)(1.136)(1)(1.095)$$

$$\frac{dD}{dT} = 16.3 \Rightarrow 16.3 = e^{a_1} \times 1.119 \Rightarrow \text{Answer: } e^{a_1} = 14.6 \text{ ft/hr}$$

In the above example, we assume a_1 to a_8 are known. In real case, we have to calculate them using "offset well data" and the multiple regression technique described by the authors.

C-2 : Describing Bit Run

If we assume : (when we don't have enough data and we have to do test, we have to do some assumptions as:

For a single bit run : ✓ formation characteristics = constant

✓ mud properties = constant

✓ Threshod Bit Weight = 0

✓ Hydraulic Impact Force (bit Hydraulic are adequate)

then $\frac{dD}{dT} = e^{a_1}(e^{a_2x_2})(e^{a_3x_3}) \cdots (e^{a_8x_8})$ becomes :

$$\frac{dD}{dT} = K e^{(a_5x_5 + a_6x_6 + a_7x_7)}$$

Substituting : $e^{a_5x_5} = e^{a_5 \ln\left(\frac{W/d_B}{4}\right)} = ((W/d_B)/4)^{a_5}$

$$e^{a_6x_6} = e^{a_6 \ln(N/100)} = (N/100)^{a_6}$$

$$e^{a_7x_7} = e^{a_7(-H)}$$

$$\boxed{\frac{dD}{dT} = K\,(W)^{a_5}\,N^{a_6}\,e^{(-H)a_7}}$$

We can evaluate the a_5 and a_6 and estimate threshold Bit Weight, by performing drilling tests over the short intervals and using information from previous bit runs.

D - Drilling Tests

Short interval tests have the following benefits:

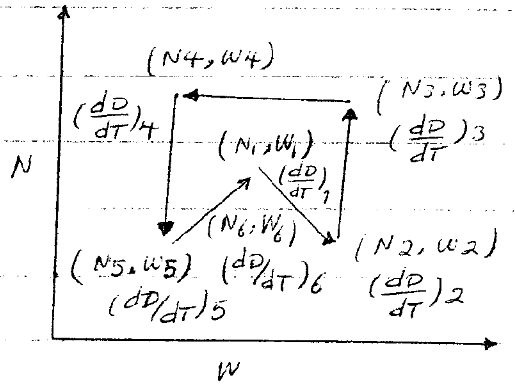
✓ It minimizes the effects of Lithology changes (which helps justifying o assigning constants values to the terms a_2 to a_4x_4 in eq. *)

✓ It minimizes the effect of tooth wear on ROP. If we use a new bit over a sufficiently short interval, the $H=0$ and term $e^{a_7x_7} = 1$.

(D-1) Five-Spot Tests

Young (1969) described a computer-controlled five spot drill rate test. which monitors penetration rate for 5-programmed combinations of bit weight and rotary speed.

- تست بیشتر در لایه‌های شیلی استفاده می‌شود. (بخاطر مقاومت زیاد لایه)



A penetration rate $(\frac{dD}{dT})_1$ is determined for the rotary speed-bit weight combination (N_1, W_1). The rig computer then commands weight and speed changes to obtain penetration 2, 3, 4, 5. point 6 serves as a control to determine if the test is acceptable. Since (N_1, W_1) is identical to (N_6, W_6) the penetration rate R_1 and R_6 should be equal within some specified tolerance (e.g, ± 15 %). The Data points 1-6 are then used to calculate average values to the constants W_0, a_6, and K. Establishing ROP for

✓(The bit weight exponent is assigned a value.).

□ Example: Five-spot drilling test

Use the data in given table to determine the threshold bit weight (W_0) and the rotating speed exponent. (a_6).

Initial and final check point should agree within 15 %.

Test Point	W (lbf)	N (RPM)	$\frac{dD}{dt}$ (ft/hr)
1	43000	115	28

2	35000	80	18
3	35000	150	27
4	50000	150	36
5	50000	80	25
6	43000	115	29

اگر تفاوت بین $\left(\frac{dD}{dt}\right)_1$ و $\left(\frac{dD}{dt}\right)_6$ به اندازه ۱۵٪ باشد، تست موفق بوده است و شرایط فوق موفق است که عوامل دیگر همه ثابت مانده‌اند و تغییری نکرده‌اند

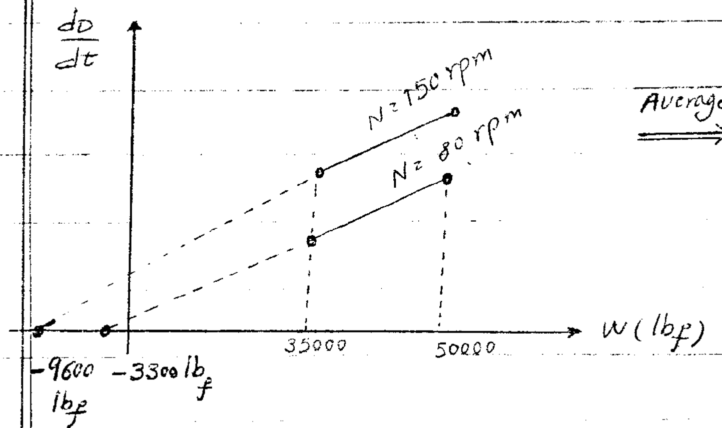
Solution:

(1) Check the agreement b/w points 1 and 6.

$\left(\frac{dD}{dt}\right)_1 = 28\ ft/hr$ $\quad\quad$ $\left(\frac{dD}{dt}\right)_6 = 29\ ft/hr$

$100 - \left(\frac{28}{29}\right) \times 100 = 3.4\ \% < 15\ \%$ (The test is ok)

(2) plot $\frac{dD}{dt}$ versus W



$\xrightarrow{\text{Average}}$ $W_o = -6450\ lb_f$

So we conclude that we have an unconsolidated rock.

(3) Determine the rotary speed exponent using a relation: $\frac{\left(\frac{dD}{dT}\right)_1}{\left(\frac{dD}{dT}\right)_2} = \left(\frac{N_1}{N_2}\right)^{a_6}$

where N_1, N_2 are rotary speed at constant weight and $\left(\frac{dD}{dT}\right)_1$, $\left(\frac{dD}{dT}\right)_2$ are corresponding penetrations.

از روی نمودار (N, W, ROP) a_6 را می‌توان بدست آورد دو نقطه‌ای

که خط افقی در یک w ثابت نمودار را قطع می‌کند

$Ln\left(\frac{36}{25}\right) = a_6\, Ln\left(\frac{150}{80}\right) \Rightarrow a_6 = 0.58$ (For bit weight 50000 lb_f)

For: 35000 $lb_f \Rightarrow a_6 = 0.645$ (For bit wt 35000 lb_f) $\Longrightarrow (a_6)_{average} = 0.61$

(D-2) Drill-off Tests:

- در حین این تست stucking محتمل است.

The Drill-off test was first proposed by Lubinski (1958), allows us to observe the relationship b/w ROP and bit weight over depth intervals using varying combination of weight and rotary speed.

$$\frac{dD}{dT} = f(\text{bit weight}, \text{Rotary Speed})$$

This test involves applying a predetermined maximum bit weight, then setting the brake and monitoring the decrease in bit weight as a function of time and constant rotary speed.

During a drill-off test, the drill string stretches as bit weight decreases and hook Load increases. The amount of stretch is equal to:

$$L = \left[\frac{0.95\,L}{E \times A}\right] \times \Delta W$$

- فرض براین است که بدلیل اصطکاک لوله حفاری فقط کشیده می شود (0.95)

where:

- upset و tool کشیده نخواهند شد (چشم پوشی خواهند شد)

- مقدار بیشینه وزن روی مته از سازنده مشخص است.

L: Length of drill pipe

E: 30×10^6 psi (206.84×10^6 KPa) for steel. (جدول الاستیسیته)

A: Cross-sectional area of drill pipe

ΔW: Change in Bit weight

This equation can be represented as: $\frac{dD}{dt} = \left[\frac{0.95L}{E \times A}\right] \frac{\Delta W}{\Delta T}$

By plotting $\frac{dD}{dt}$ vs. W on Log-Log paper, we can obtain a straight Line having a slope equal to bit weight exponent, This assuming $W_c = 0$, as in the case for soft formation. (اگر مقدار $W_c=0$ باشد خط راست بوجود می آید این برای سازندهای سست است)

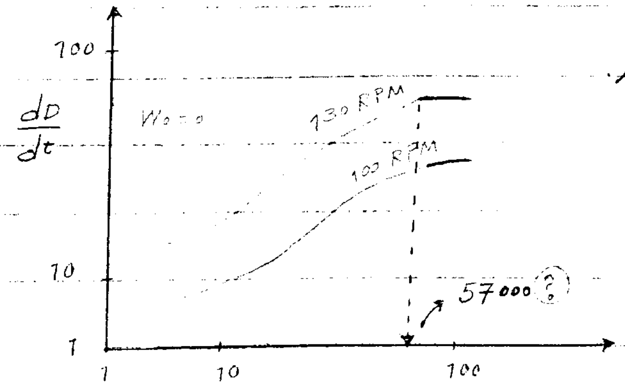
□ **Example:** The following information was recorded for drill-off test. Determine the bit weight exponent and rotary speed exponent.

(از جدول)

N = 100 RPM			N = 130 RPM			N = 100 RPM	N = 130 RPM
W (lbf)	time (min)	Δt (s)	W (lbf)	Δt	W_{av}	$\frac{dD}{dt}$ (ft/min)	$\frac{dD}{dt}$ (ft/min)
70000	0:0	0	70000	0	68000	23	25.1
66000	0:35	35	66000	32	64000	22.3	25.1

62000	1:11	36	62000	32	60000	20.1	23.7
58000	1:51	40	58000	34	56000	18.3	23.0
54000	2:35	44	54000	35	52000	(16.1	20.1)
50000	⋮	⋮	⋮		⋮	⋮	⋮
⋮						10.05	11.2
34000	8:12	84	34000	72	32000	7.5	9.3
30000							

ـ وقتی drill-off test انجام شود دیگر five-spot test
نیازی به انجام نیست چون هر دو پارامتر N و WOB را تعیین می‌زند.



Conditions prior to test : depth 9750 ft

Drill pipe : 9300 ft of 5", 19.5 lb/ft (ID = 4.276 in)

Bit type and size : $12\frac{1}{4}$ in , SUH (IADC Code 215)

$\frac{dD}{dT} = 15$ ft/hr , $W = 50000$ lbf , $N = 100$ RPM

Test Data : Characteristics Time : 50000 lbf ⇒ 45000 lbf in 105 sec.

Initial Test Weight : 70000 lbf

Test was run at $N = 100$ RPM and $N = 130$ RPM.

Solution :

$$\frac{dD}{dT} = \left(\frac{0.95L}{E \times A}\right)\left(\frac{\Delta W}{\Delta T}\right) \quad , \quad A = \frac{\pi}{4}\left(5^2 - 4.276^2\right) = 5.275\ in^2$$

$$\frac{dD}{dT} = \left(\frac{0.95 \times 9300}{30 \times 10^6 \times 5.275}\right)\left(\frac{7000 - 3000}{\Delta T}\right) \Rightarrow \frac{dD}{dt} = \frac{804}{\Delta T}$$

ft/hr ? seconds

$$\left(\frac{dD}{dT}\right)_1 \Big/ \left(\frac{dD}{dT}\right)_2 = \left(\frac{N_1}{N_2}\right)^{a_6} \Rightarrow a_6 = \frac{\log\left[\left(\frac{dD}{dt}\right)_1 \Big/ \left(\frac{dD}{dt}\right)_2\right]}{\log\left(N_1/N_2\right)} \quad (\text{at } W = cte)$$

$$a_6 = \frac{\log(20.1/16.1)}{\log(130/100)} = 0.85$$

How to obtain a_7 ?

$$a_5 = \frac{\log(23.0/20.1)}{\log(52000/48000)} = 1.68 \quad \text{or} \quad a_5 = \frac{\log(23/20.1)}{\log(56000/52000)} = 1.81$$

Vidder / Chevron ; developed a variation of the drill-off test that involves the following procedure:

(1) Select a test depth that is reasonably certain to be in a uniform shale section.

(2) Drill for some time at the current bit weight to establish the bit's bottom-hole pattern.

(3) With conditions stablished, Lock the brake and record the characteristic time required to drill-off 10 % of the current bit weight.

(4) Increase the bit weight to its initial test value of at least 20 % above the current bit weight. Drill long enough at this weight to establish a new bottom-hole pattern. This time is equal to one increment of characteristics time for each 10 % increase in bit weight. (e.g, raising the bit weight from 50000 lbf to 60000 lbf represents a 20 %. increase, if the characteristic time recorded in step 3 was two minutes, we would drill at 60000 lbf for four minutes)

H.w : World Oil also lists in Drill Bit Classifier. (published Annually)

Cost per drilled foot:

As demonstrated in an example before, maximum Rop do not always result in minimum cost drilling. The footage and bit life parameters that result from various weights and Rotary speed must be substituted into the cost per foot equation to provide a true indication of optimum conditions. The last item - cost per drilled foot - is the most important criteria for establishing drilling parameters.

con'd from P. 31 :

Recalculate the sizes for two nuzzles: $0.12 = \frac{2\pi}{4}\left(\frac{d}{32}\right)^2 \Rightarrow d = 8.8$

Use two $9/_{32}$ nds nozzles at 175 gal/min.

✓How to obtain a_7:

Bourgoyne and Young (1974) suggest evaluating observed declines in penetration rate with tooth wear based on bit grading, for previous bits run under identical condition.

□ **Example**: The initial penetration rate for a bit, drilling in the same interval that was tested in the previous example was 24 ft/hr. The previous bit was of the same size and type and was graded T-7, after drilling in the same formation using identical and constant bit weight, rotary speed, hydraulic, mud properties, The ROP of current bit was 13 ft/hr, just before pulling determine tooth wear constant a_7.

Solution: $\frac{dD}{dT} = K \times W^{1.6} \times N^{0.55} \times e^{(-H)\,a_7}$

Initial Rate (new bit, $H=0$) $\Rightarrow$ $24\ ft/hr = K\,W^{1.6} \times N^{0.85}$ (1)

$$13\ ft/hr = K\,W^{1.6} \times N^{0.85} \times e^{-(0.875)\,a_7} \quad (2)$$

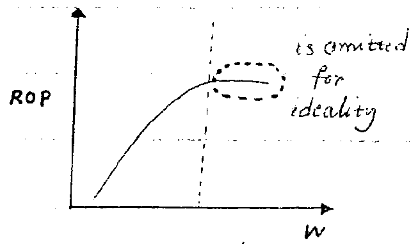
Deviding Eqn. (2) by (1) $\Rightarrow \frac{13}{24} = e^{-(0.875)\,a_7}$

$$\Rightarrow a_7 = 0.7$$

For the condition described we may express our drilling rate eqn. as:

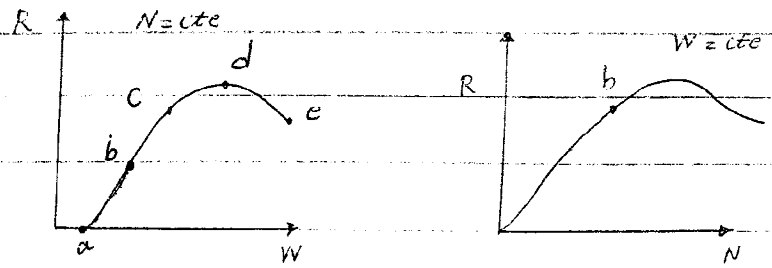
$\frac{dD}{dT} = K \times W^{1.6} \times N^{0.85} \times e^{0.7(-H)}$ - این معادله شکل معادله بینگهام برای انتخاب مته به کار می رود.

We did not mention the optimum W and N, but we show in the test the limitation of floundering.



(5) Constraint on Bit weight and Floundering

It is interesting to note that in the previous example the optimum R did not occur under conditions of maximum bit weight and rotary speed. Instead the bit floundering including inefficient cutting removal and illustrating the limit on bit weight and rotary speed. [illegible]



"معضل کردن"

✓Hydraulic : R must not overwhelm our circulating system ability to clean the hole and conditions the mud. also it must not be high as to make Kick detection difficult.

✓Hole Deviation : Excess bit weight may cause dog legs or other crooked hole problems (spending more money). It can be prevented by having Light BHA. (using stabilizer). ✓Crooked Hole Problems : Dog Leg, Key Seat, Bit Gauge Wear.

□ Bit Specifications :

Bit manufacturers design their product to run within certain range of bit weight and rotary speed. (e.g. 1000 – 5000 psia or 70 – 120 RPM)

Cost per drilled foot : $\#/ft = \dfrac{C_B + C_R(t+T)}{\Delta D}$

We know that $\#/ft \propto \dfrac{1}{ROP}$, but the Limitation is bit Life. ($\dfrac{dD}{dt} \Rightarrow$ Bit Life)

It's not important how fast we can drill, it is important how Long with the most rapidness we can drill.

✓The more you are in the hole with a single bit, then you can minimize the cost. A primary concept regarding bit weight and rotary speed as they relate to the minimum drilling costs is the "bit life." It does little good to maximize ROP with high WOB and high N, if the result is premature bit wear, extra trip time or Lost cones.

(6) Bit Life

The life of a rolling cutter bit depends on two parameters : (1) tooth wear
(2) Bearing wear

Cutter life directly affects ROP as shown: $\frac{dD}{dT} = K(w)^{a_5} N^{a_6} e^{-(H)a_7}$

As the term indicates (e^{-Ha_7}) tooth wear is a gradual process. By contrasts bearing failure results from fatigue «دوره‌ای» and occurs much more abruptly as evidenced by the characteristics "torquing up" of bit.

While bearing life does not directly effect ROP, it can bring a sudden end to a bit run.

✓ Bit life estimates depend on determining whether the tooth or bearing failure is most likely to occur first.

As a result, a rule of thumb, bearing failure is the deciding parameter in soft, non-abrassive formations, where bit cutter experience little wear. In hard abrassive formations, on the other hand, the bit teeth tend to wear out before the bearing do.

6-6:1 : Tooth Wear

Drilling parameters that affect wear include: Rotary speed
Bit Weight
Tooth wear rate ($\frac{dh}{dT}$)

Rotary Speed

The rate of tooth wear for a given milled-tooth bit increases with increasing N, as indicated by the following equation. (Young, 1969)

$$\frac{dh}{dT} = PN + QN^3$$

where:

P, Q : empirical constants, based on bit type

N : Rotary Speed

Bit Weight

The rate of tooth wear increases "non-Linearly" with increasing bit weight

This increase is due to the sever chipping and eventual tooth destruction that occur above some maximum weight. we may express the tooth wear bit wt relationship as follows : $\frac{dh}{dt} = \frac{1}{-D_1W+D2}$

where : D_1 and $D2$ are constants based on bit size , W = bit weight .

Tooth Wear Rate :

The rate of which wear occurs is proportional to the cross-sectional area that the tooth presents to the formation. because tooth wear is roughly pyramidal in shape in cross-sectional area of the tooth.

✓Cutting surface increases as wear causes the tooth height to decrease. Therefore the rate of wear decreases as the tooth gets «فرسوده، ضعیف» dull , Hence :

Young (1969) expresses as : $\frac{dh}{dT} \propto \frac{1}{1+C_1H}$

where:

C_1 : constant , based on tooth shape , the type of hardfacing on the tooth and the degree of heat treating used in

H: fraction of tooth height that has been worn away.

Combined effects of rotary speed , bit weight , Young has shown that:

$$\frac{dh}{dT} = 0.001 \times A_f \times \frac{PN+QN^3}{(-D_1W+D_2)} \times \frac{1}{(1+C_1H)}$$

where ;

A_f is an abrassiveness constant for the given formation.

the abrassiveness constant (A_f) is an indicator of formation characteristics as they relate to tooth wear. An A_f value b/w 0 to 4 indicate Low abrasiveness and suggests bearing wear on the bits failure mechanism.

An A_f range of 5-10 indicates high abrasive tendencies and suggests that tooth wear is primary failure mechanism. note that in the values b/w 4-5 are not diagnostic of the failure made. by integrating above eqn. (Limits of integration are Last and initial Limit of the bit) , hence :

~~Drilling Engineering (2) (P. 49)~~

$$A_f = \frac{1000(-D_1 W + D_2)}{(PN + QN^3)T} \times \left[H_f + \left(\frac{C_1}{2} \times H_f^2 \right) \right]$$

where H_f : final Wear fraction of tooth height.

✓ tooth failure corresponds to complet wear (i.e $H_f = 1$), the time at which this occures is equal to :

(بیشترین زمانی که مته را می‌توان در چاه نگه داشت)

$$\boxed{T_{max} = \frac{1000(-D_1 W + D_2)}{A_f (PN + QN^3)} \left(1 + \frac{C_1}{2} \right)}$$

tooth wear parameters for 3-cone rock bits (After Young, 1969)

✓ milled tooth
✓ Insert tooth

IADC Group	P	Q	C_1
1-1 to 1-2	2.5	1.008×10^{-4}	7
1-3 to 1-4	2.0	0.870×10^{-4}	6
2-1	1.5	0.653×10^{-4}	5
2-2 to 2-3	1.2	0.582×10^{-4}	4
2-4	0.9	0.392×10^{-4}	3
3-1	0.65	0.283×10^{-4}	2
3-2 to 2-4	0.5	0.218×10^{-4}	2
4	0.5	0.218×10^{-4}	2

Bit Size parameters for three-cone bit (Rock bit) values based on a destruction wt of 1000 lbf/in. The values for D_1 and D_2 apply where bit weight is expressed in 1000-lbf.

Bit OD (in)	D_1	D_2
6.250	0.088	5.5
6.750	0.088	5.67
7.785	0.074	5.94
8.625	0.071	6.11
9.625	0.066	6.38
9.875	0.065	6.44
10.75	0.062	6.88
12.25	0.058	7.15

Example:

Estimate the Abrasiveness of constant and determine the most Likely mode of bit failure, given the following data from previous bit run. Assume constant bit types, formation properties, operating conditions and rotating hrs from both the current and the previous run.

If tooth failure is the determining failure criterion, estimate the time at which it occures.

Rotary Speed: 60 RPM, WOB: 40000 lbf, Bit: 9 7/8 in, IADC code 214, Rotating Hours: 12, Tooth Grade: T6 (H = 0.75)

Solution:

$$A_f = \frac{1000\,(-D_1 \overset{40}{W} + D_2)}{(PN + QN^3)\,T}\left[H_f + \left(\frac{C_1}{2} \times H_f^2\right)\right]$$

$D_1 = 0.065$ $\boxed{A_f = 6.6}$ $10 > A_f > 5$ (tooth wear occurs first)

$D_2 = 6.44$

$P = 1.5$

$Q = 0.653 \times 10^{-4}$

$C_1 = 5$

for the conditions described, tooth wear is the determining failure criteria, tooth failure occurs when the rotating hrs are equal to:

$$T = \frac{1000\,(-D_1 W + D_2)}{A_f\,(PN + QN^3)}\left(1 + \frac{C_1}{2}\right)$$

$$T = \frac{1000\,(-0.065 \times 40 + 6.44)}{6.6\left[(1.5 \times 60) + (0.653 \times 10^{-4})\,60^3\right]}\left(1 + \frac{5}{2}\right) \Rightarrow \boxed{T = 19.6 \text{ hours}}$$

6.6.2 Bearing Wear

If we determine that tooth wear is not the critical bit life parameter (i.e. $0 < A_f < 4$) we then need to Look at the bearings.

Bit weight: The rate of bearing wear increases rapidly and non-Linearly with increasing bit weight, we can describe this relationship as: $\frac{dB}{dT} \propto W^6$ (Young, 1969)

Where: $\frac{dB}{dT}$ = instantaneous rate of bearing wear

W = bit weight, 1000 lbf

b = bearing wear exponent (typically equal to 1.5)

B عددی شاخص از نحوه عمل bearing است که اگر Bearing کاملاً نو باشد B=0 است و اگر Bearing در حال ع شدن باشد B=1 خواهد بود. ($0 \leq B \leq 1$)

Rotary Speed:

Bearing Life generally depends on the total number of times a bit turns, regardless of the rate at which it turns. This means that the rate of bearing wear increases Linearly with increasing Rotary Speed: $\frac{dB}{dT} \propto N$

Combined Effects:

$$\frac{dB}{dT} = \left(\frac{1}{b}\right) \times N W^{1.5}$$

Where:

b = bearing wear constant

✓ We may determine the bearing wear constant from historical drilling data. Its values depends on the mud properties, bit size, bit type.

Integrating this eqn. over the Life of the bearings and then solving for b, we obtain:

$$b = \frac{N W^{1.5} \times T}{B}$$

Where B: Final bearing grade expressed as decimal (e.g, if IADC bearing grade is B-6 then B = 0.75)

✓ If we have data from previous bit run, where bit type, formation property and other operating conditions are identical to the current bit run, we can calculate the bearing wear constant and estimate the bearing Life for the current bit.

$$T = \frac{b \times B}{N W^{1.5}}$$

✓ This procedure removes some of the guesswork involved in estimating bit Life, and reduces our chances of either pulling a "green" bit (one with bearing still in good condition or Losing cones in the hole.)

Example: Estimating bearing life

Given the following data, from the previous run, and assuming no change in operating conditions, determine the current bit will experience complete bearing failure:

$A_f = 2.0$ Grade: $T4-B6-1$

$N = 110$ RPM $W = 55000$ lbf

$T = 25$ hrs $B = 0.75$ برابر $(T4-B6.1)$ و جدول $T = 25$ از زمان previous run برای B مقدار خواهد بود.

Solution:

From previous run: $b = \dfrac{N w^{1.5} T}{B} = \dfrac{110 \times 5.5^{1.5} \times 25}{0.75} \Rightarrow b = 1,495,600$

For Current bit run:

$$T = \frac{1495600 \times 1.0}{110 \times 55^{1.5}} \Rightarrow \boxed{T = 33.3 \text{ hrs}}$$

Bearings: (1) Sealed Bearing — بیرینگ‌ها در تماس با گل حفاری نیستند و داخل مخزن روغنی هستند.

(2) non-sealed Bearing — بیرینگ‌ها در تماس مستقیم با گل حفاری هستند و فقط میانشان گریس کاری شده است.

Effect of Bit Type:

Bearing life also depends on whether the bearing has a non-sealed or sealed Lubricating system. non-sealed system have no mechanical means of preventing mud entering into the bearing assembly. The bearings are packed with a high-viscosity grease as a Lubricant. While the grease may inhibit mud entry, it does not protect the bearing as a sealed bearing bit which is so-called because of the seal that is placed b/w the back of the cone and interior of the bit Leg.

Field data have indicated that this seal assembly, by preventing mud entry, can increase bearing life by an estimated 30% over that of non-sealed bearing bits. Bearing capacity varies among bit types, based on their design for hard or soft formations. Table below shows relative bearing capacities for several bit types, where a value of 1.0 is the relative bearing capacity of a bit for soft, Low-compressive strength rock.

IADC code	Bearing Capacity
1,1,1	1.0
1,2,1	1.15
1,3,1	1.20
2,1,1	1.35
2,2,1	1.45
3,1,1	1.45

Example: Bearing life calculation for sealed bearing bit

Using the data from previous example, estimate the bearing life of a 1,3,1 bit run with 55000 lbf bit weight at 110 RPM.

Solution: $b = 1495600$

modify b to reflect the increased bearing capacity:

$1,495600 \times 1.2 = 1794700$

For current bit run: $T = \dfrac{1,794700 \times 1.0}{110 \times 55^{1.5}} \Rightarrow \boxed{T = 40 hrs}$

(3) Bit Selection

The previous bit run is our primary guide line in bit selection. This is assuming of course that the next interval that we drill exhibit the same bit wear characteristics as the previous interval:

✓ If the abrasiveness factor is less than 4, a longer tooth softer formation bit should probably be used for the next run.

✓ If the abrasiveness factor is greater than 4, a tungstan carbide insert bit may be preferable to a milled tooth bit.

✓Sealed bearing bits are usually a cost-effective option for bit selection. Although more expensive than non-sealed roller bearings; they offer greater bearing capacity and Larger bit runs.

Note: As technological advances and increased competition among bit manufacturers gives rise to an over-growing number of rolling-cutter and fixed-cutter bit types. bit selection becomes increasingly important in the overall ask of optimization.

(4) Predicting Bit Performance:

If the bit type has changed from the previous run we need to recalculate the bearing and tooth wear constant and make a new estimate of bit Life based on the current interval having the same drillability as the previous interval.

6.7: Implementation Procedures.

Although the relationships among drilling parameters are complex, our efforts to describe them come back to a basic objective: to determine what combination of operating conditions results in minimum cost drilling. Once we know bit rotating times and footage for various combinations, of weight and rotary speed, it is a relatively straithforward matter to use the cost per foot relationship to determine which of these combinations is most favorable.

✓Optimization procedures should be practical and simple enough to implement at the rig on a daily basis, and should include the following basic steps:

(1) Data Gathering

(2) Bit Performance Evaluation

(3) Bit Selection

(4) Predicting Performance

(5) Estimating Optimum Weight and Rotary Speed.

(1) Data Gathering

We should begin by collecting as much information as possible about drilling conditions. This involves:

✓Review the drilling contract and analyzing daily drilling expenses to determine an equivalent hourly rig cost.

✓Recording bit performance informations, including footage, rotating hours and trip times.

✓Accurately grading pulled Bit.

✓Conducting short-interval drilling tests during the current bit run, to determine the formation's bit weight exponent, rotary speed exponent and threshold bit weight and correlating this information with offset well data, if available.

2. Bit Performance Evaluation

Based on information from previous bit run, we can determine the whether tooth wear or bearing wear is the defining bit failure criterion and estimate bit life and footage.

✓We can summarize these steps as follows:

(a)

(b) If $A_f > 5.0$, Use tooth failure Criterion:

$$T = \frac{1000\,(-D_1 W + D2)}{A_f\,(PN + QN^3)}\left(H_f + \frac{C_1}{2} H_f^2\right)$$

(c) If $A_f < 4.0$, Calculate the bearing constants, and use bearing failure Criterion: $b = (N w^{1.5} T / B)$

(d) If $4.0 < A_f < 5.0$, Assume Bearing Failure unless additional evidence indicates otherwise.

5. Establishing Optimum Weight and Rotary Speed

We can insert the estimated values for bit life and footage at various weights and rotary speeds into the cost equations and select the optimum conditions based on minimum cost drilling.

Blow-Out Prevention and Well Control

Rotating BOP : In UBD

Drill spool : دسترسی به Annular

Choke Line ↙ ↘ Kill Line ⇒ Drill spool بستگی به نوع استفاده از

Snapping حتی با بستن Annular هم ممکن است فشار
ناگهانی لوله حفاری را پرتاب کند.

Blind Ram (صفحه از یک طرف) در چاه که لوله نداریم چاه را مسدود می کند.
Shear Ram (صفحه دو طرفه) در چاه با لوله حفاری، لوله را قطع می کند.

Fig. 20 : با چاقو بریده می شود تا گل دور Annular را بگیرد و جمع شود.

✓ Causes of Abnormality :

Underground Blow-out مثلاً لایه شیلی، لایه ماسه سنگی را احاطه کرده و لایه ماسه سنگی تحت
فشار است چون در خلال مهاجرت، آب شیل جابجا شده است.

Drilling Break وزن Hook زیاد می شود (وارد سازندی می شویم که تخلخل آن
زیاد است) یعنی وزن روی مته کم می شود.

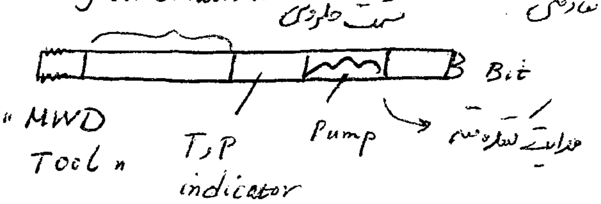
Gas Cut Mud ورود گاز به گل حفاری در سطح

(تغییر لایه سازند) از نشانه‌های Kick تغییر سایز خرده‌ها است.

در لایه‌های شیلی خرده‌ها ورقه‌ای هستند.

Logging از نمودار مقاومتی مقدار Sw محاسبه شده و مقدار تخلخل
پیش بینی خواهد شد.

Down-hole meter → MWD / LWD → اندازه‌گیری فشار و دمای سازند
Log Combinations اندازه‌گیری‌های مقاومتی نفوذ، سونیک، مقاومتی



در سمت حلزونی، گل آن را می چرخاند و بنابراین سمت حلزونی
مته را می چرخاند.

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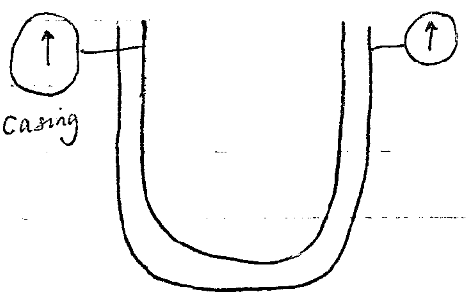
Note: Lithology changes ⇒ Abnormal pressure

$q = J \, \Delta P$

Rate of Kick

MWD بیشتر در جاهایی استفاده می شود که می خواهیم انحراف در حفاری ایجاد کنیم.



اگر یکی از دو فشار زیاد شود
نشانه وجود (ورود)
سیال اضافی است (Kick)

در موقع حفاری Choke Line نیمه بسته است (برای ایمنی) ← اما اکثر مواقع بسته است
← در لایه هایی که احتمال وجود گاز است، Gas Kick احتمالی خواهیم داشت.

Packers: Mechanical Packer

Permanent Packer

Hydraulic Packer

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OILWELL DRILLING ENGINEERING

PRINCIPLES AND PRACTICE

H. RABIA

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Chapter 5

Fundamentals of Fluid Flow

This chapter aims to give a brief introduction to the fundamentals of fluid flow in pipes and annuli. The discussion will be limited to Bingham plastic and power-law models, these models being the most widely used in the drilling industry. Full details of the derivation of the various equations are given to help the reader appreciate the limitations of these equations.

This chapter will cover the following topics:

Fluid flow
Viscosity
Types of flow
Criteria for type of flow
Types of fluid
Viscometers
Derivation of laminar flow equations
  Bingham plastic model
  Power-law model
Turbulent flow
Flow through nozzles

FLUID FLOW

A fluid flowing along a conduit of any cross-section has a stationary layer adjacent to the conduit wall. The velocity of this layer is zero and the velocities of adjacent layers increase progressively until a maximum velocity is attained at the centre of the conduit, as shown in Figure 5.1.

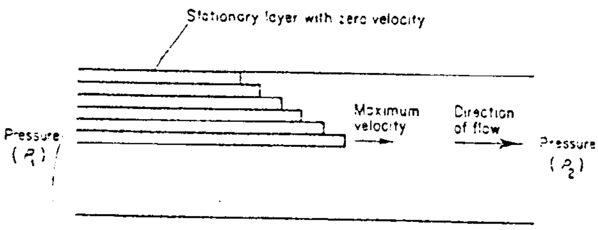

Fig. 5.1. Fluid flow through a pipe.

This progression from zero velocity at the pipe wall to maximum velocity at the centre of the conduit results in the sliding of layers past one another; a high-velocity layer slides past its adjacent low-velocity layer, and so on. Fluid flow is, therefore, a result of this sliding action and, in order to maintain this flow, a continuous supply of energy is required. For example, a pump is required to lift water from a deep well to a surface tank.

The sliding action of fluid layers is accompanied by shear stress (or frictional drag) which is highly dependent on the velocity and viscosity of the fluid.

VISCOSITY

Viscosity is a property which controls the magnitude of the shear stress that develops when one layer of fluid slides over another. Viscosity is, therefore, a measure of the strength of the internal resistance offered by the cohesive forces between the fluid molecules when motion is induced. Viscosity is also dependent on the type and the temperature of fluid. Temperature largely affects the intermolecular distances. For liquids the distance between the molecules is increased with increasing temperature, which reduces the magnitude of the cohesive forces and, in

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turn, the fluid viscosity. For gases the increased temperature causes the vibrational forces of the molecules to increase and the cohesive forces to decrease. In practice, the vibrational forces of gas exceed the cohesive forces, which results in increased viscosity with increasing temperature.

For a drilling mud composed of water and solids the viscosity is controlled by quantity, size and shape of solids.

The viscosity of fluids may be related to measurable parameters by considering the deformation of an elemental cube as shown in Figure 5.2. The cube is subjected to a force, F, applied parallel to the surface labelled 1, having a cross-sectional area A. The resulting shear stress, τ, is given by

$$\tau = \frac{F}{A} \qquad (5.1)$$

This shear stress results in deformation of the fluid layer from a cubic shape (Figure 5.2a) to a rhombic shape, as shown in Figure 5.2(b). This deformation is analogous to the elongation or strain in elastic solids. Fluid deformation is referred to as shear strain and is described by the ratio between the velocity difference between the top and bottom of the deformed cube and the height of the cube.

Hence,

shear strain $$\gamma = \frac{(V + dV) - V}{dr}$$

$$= \frac{dV}{dr}\,s^{-1} \qquad (5.2)$$

Experimental evidence indicates that τ is related to γ either linearly or non-linearly. Fluids exhibiting a linear relationship between τ and γ are referred to as Newtonian fluids and, for this type, viscosity, μ, is expressed as

$$\tau = \mu\gamma \qquad (5.3)$$

Substitution of Equations (5.1) and (5.2) in Equation (5.3) gives

$$\frac{F}{A} = \mu\left(\frac{-dV}{dr}\right) \qquad (5.4)$$

The negative sign is included to indicate that velocity decreases away from the centre as the distance, dr, increases, to allow for the fact that a stationary layer exists at the pipe wall. The reader should note that the term 'viscosity' in Equations (5.3) and (5.4) and the rest of the book refers to dynamic viscosity.

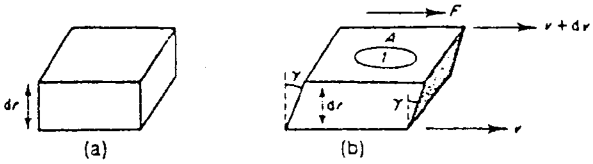

Fig. 5.2. Elemental cube of fluid: (a) before shear; (b) after shear.

Units of viscosity

In both Imperial and metric units viscosity is normally expressed in units of poise (P) or centipoise (cP).

The consistent units of viscosity in the Imperial system are

$$\frac{\text{lb s}}{\text{ft}^2} \text{ or } \frac{\text{lbm}}{\text{ft s}}$$

In metric units, from Equation (5.4),

$$\mu = \frac{F/A}{dv/dr} = \frac{\text{N/m}^2}{\left(\frac{\text{m/s}}{\text{m}}\right)}$$

$$= \frac{\text{N s}}{\text{m}^2}$$

$$= \frac{(\text{kg m/s}^2)\,\text{s}}{\text{m}^2} = \frac{\text{kg}}{\text{m s}}$$

$$= \frac{\text{g} \times 10^3}{\text{cm} \times 10^2\,\text{s}} = \frac{10\,\text{g}}{\text{cm s}}$$

By definition,

$$1\ \text{P} = 1\,\frac{\text{g}}{\text{cm s}}$$

and 1 P = 100 cP. Hence,

$$1\ \text{cP} = 10^{-3}\,\frac{\text{kg}}{\text{m s}}$$

Also,

$$1\ \text{cP} = 2.0886 \times 10^{-5}\,\frac{\text{lb s}}{\text{ft}^2}$$

Or

$$1\ \text{cP} = 6.719 \times 10^{-4}\,\frac{\text{lbm}}{\text{ft s}}$$

Hence

$$1\ \text{P} = 2.089 \times 10^{-3}\,\frac{\text{lb s}}{\text{ft}^2}$$

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and

$$\frac{\text{lb s}}{\text{ft}^2} = 47\,886 \text{ cP}$$

TYPES OF FLOW

Generally speaking, two types of flow can be recognised: laminar flow and turbulent flow.

Laminar flow

In laminar flow the flow pattern is smooth, with fluid layers travelling in straight lines parallel to the conduit axis. The velocity of each layer increases towards the middle of the stream until some maximum velocity is reached.

In laminar flow shear resistance is caused by the sliding action only and is independent of the roughness of the pipe. Laminar flow develops at low velocities and there is only one component of fluid velocity: a longitudinal component.

A special type of laminar flow with a flat centre portion is called a plug flow (Figure 5.3). In the flat portion there is no shear of fluid layers, and this is the reason that they are moving at the same velocity. It should be observed that plug flow occurs only with yield stress materials.

In oil-well drilling plug flow occurs at low velocities and when the mud thickness (viscosity) is large.

Turbulent flow

In turbulent flow the flow pattern is random in both time and space. The chaotic and disordered motion of fluid particles in turbulent flow results in two components of velocity: a longitudinal and a transverse component. The longitudinal velocity attempts to make the fluid flow parallel to the conduit axis, while the transverse component attempts to move the fluid in a direction normal to the pipe axis.

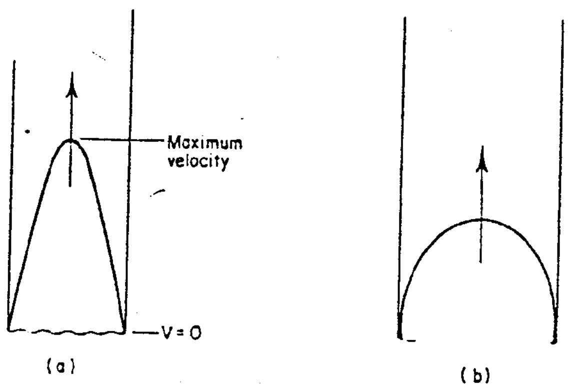

Fig. 5.3. (a) Laminar flow; (b) plug flow.

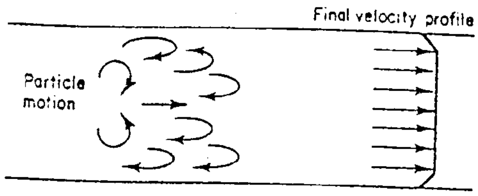

Fig. 5.4. Turbulent flow.

The motion of particles in a direction normal to the longitudinal direction generates another shear resistance in addition to the laminar shear resistance. As previously discussed, the laminar shear resistance develops as a result of the sliding of one layer over another. In a fully developed turbulent flow the shear resistance due to turbulence can be many times the laminar shear resistance.

Despite turbulence, the final velocity profile tends to be a uniform one (Figure 5.4); this is largely attributed to the mixing of fluid particles which leads to interchange of momentum between high-velocity and low-velocity particles, which gives rise to a fairly flat profile.

Even in turbulent flow, particle fluctuation near the conduit wall dies out and the flow pattern in this region is essentially laminar. This region is normally called the laminar sublayer, and its thickness depends on the degree of turbulence. The relationship between the thickness of this laminar sublayer and the degree of turbulence is an inverse one.

In oil-well drilling turbulent flow is to be avoided as far as possible, since turbulence can cause severe hole erosion. Pressure losses also increase with degree of turbulence. However, in cementing turbulence is deliberately initiated to help to displace the mud cake from the walls of the hole, thus allowing the cement to contact the fresh surfaces of the formation. This will then result in a better cement job, as will be discussed in Chapter 11.

CRITERIA FOR TYPE OF FLOW

From the previous discussion it is apparent that in laminar flow shear resistance is dependent solely on the sliding action of layers. In turbulent flow the additional turbulent shear resistance is dependent on the magnitude of the transverse velocity. Thus, it is convenient to use fluid velocity as a criterion for determining the type of flow. Two other fluid pro-

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perties, namely viscosity and density, can also be used in conjunction with velocity and conduit diameter to determine the type of flow. These parameters are grouped to form a dimensionless number called the Reynolds number, (Re):

$$(Re) = \frac{DV\rho}{\mu} \tag{5.5}$$

where D = conduit diameter; V = fluid velocity; ρ = fluid density; and μ = fluid viscosity.

It has been experimentally established that at a certain critical value of (Re) the flow pattern changes from laminar to turbulent. The magnitude of this critical value depends on many factors, including pipe wall roughness, viscosity of fluid and proximity of vibration. In most applications, however, fully turbulent flow develops at (Re) values of greater than 3000. For (Re) values of less than 2000, the flow is always laminar. In the transitional flow, where Re is between 2000 and 3000, the flow is often described as 'plug flow'. In plug flow a central portion exists where shear resistance is zero (as shown in Figure 5.3).

Reynolds number using field units

Parameter	*Metric unit*	*Imperial unit*
Diameter	mm	in
Velocity	m/s	ft/min
Density	kg/l	lbm/gal
Viscosity	cP	cP

Metric units:

$$(Re) = \frac{DV\rho}{\mu}$$

$$\text{Constant} = \frac{\left(\text{mm} \times \frac{1\ \text{m}}{10^3\ \text{mm}}\right)\left(\frac{\text{m}}{\text{s}}\right)\left(\frac{\text{kg}}{\text{l}} \times \frac{1000\ \text{l}}{\text{m}^3}\right)}{\left(\text{cP} \times \frac{10^{-3}\ \text{Kg/m s}}{\text{cP}}\right)}$$

$$(Re) = \frac{1000\,DV\rho}{\mu} \tag{5.5a}$$

Imperial units:

$$(Re) = \frac{DV\rho}{\mu}$$

Constant

$$= \frac{\left(\text{in} \times \frac{1\ \text{ft}}{12\ \text{in}}\right)\left(\frac{\text{ft}}{\text{min}} \times \frac{\text{min}}{60\ \text{s}}\right)\left(\frac{\text{lbm}}{\text{gal}} \times \frac{7.48\ \text{gal}}{1\ \text{ft}^3}\right)}{\text{cP} \times \frac{\left(2.0886 \times 10^{-5}\ \frac{\text{lb s}}{\text{ft}^2}\right)}{\text{cP}}}$$

$$\left(Pa.s \rightarrow \frac{lb_f.s}{ft^2}\right)$$

$$= 497.4092\ \frac{\text{ft.lbm}}{\text{s}^2.\text{lb}}$$

$1\ lb = 0.452\ Kg$

$1\ ft = 0.3048\ m$

$$(Re) = 15.46\,\frac{DV\rho}{\mu} \tag{5.5b}$$

$\left(\frac{g}{g_c}\right)_{SI} = 9.806$

$\left(g/g_c\right)_{Bus} = 1$

TYPES OF FLUID

Newtonian fluid

A Newtonian fluid is defined by a straight-line relationship between τ and γ with a slope equal to the dynamic viscosity of the fluid, i.e. $\tau = \mu\gamma$. In this type of fluid, viscosity is constant and is only influenced by changes in temperature and pressure (as shown in Figure 5.5). Examples include oil and water.

Non-Newtonian fluid (Drilling mud and Cement slurries)

A non-Newtonian fluid is a fluid that does not show a linear relationship between τ and γ, i.e. μ is not a constant. The viscosity of this type of fluid is proportional to the magnitude of shear stress or the duration of shear. Examples include drilling mud and cement slurries.

In general three major types of non-Newtonian fluid can be recognised.

Bingham plastic fluid (time-independent)

In a Bingham plastic fluid, deformation takes place after a minimum value of shear stress is exceeded. This minimum value is referred to as the yield stress or

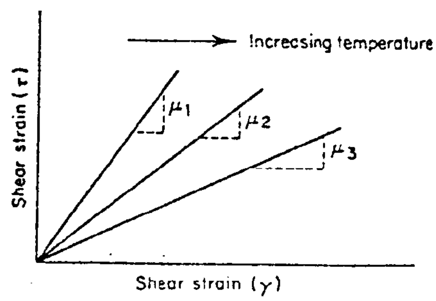

Fig. 5.5. Newtonian fluid.

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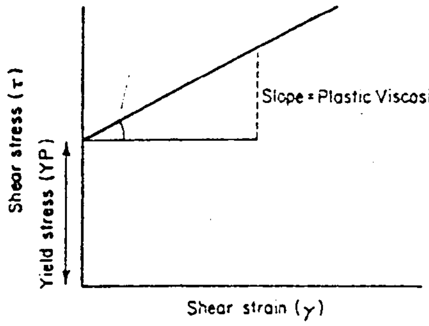

Fig. 5.6. Bingham plastic flow.

'yield point' (YP) (Figure 5.6). Beyond YP the relationship between τ and γ is linear, with a constant value of viscosity known as plastic viscosity (PV). Plastic viscosity is, again, dependent on temperature and pressure.

Hence, for a Bingham plastic fluid,

$$\tau = YP + (PV)\,\gamma$$

or

$$\tau = YP + (PV)\left(\frac{-dv}{dr}\right) \qquad (5.6)$$

The yield point or yield stress is normally measured in lb/100 ft^2 using a viscometer, as detailed in Chapter 6. In metric units the yield point is expressed in N/m^2.

Power-law fluid (time-independent)

In a power-law fluid τ and γ are related by the following expression:

$$\tau = K(\gamma)^n$$

or

Consistency of fluid

تاثیر درجه غیرنیوتنی

توان سیال

$$\tau = K\left(\frac{-dv}{dr}\right)^n \qquad (5.7)$$

where n = flow behaviour index, which varies between 0 and 1; and k = consistency index. A power-law fluid is shown graphically in Figure 5.7.

It should be noted that when $n = 1$, Equation (5.7) reduces to

$$\tau = K\gamma$$

where $K = \mu$, and the relationship reduces to that of a Newtonian fluid. The value of n gives an indication of the degree of non-Newtonian behaviour, and k refers to the consistency of the fluid. Large values of k mean that the fluid is very thick.

تعداد برخورد در واحد زمان

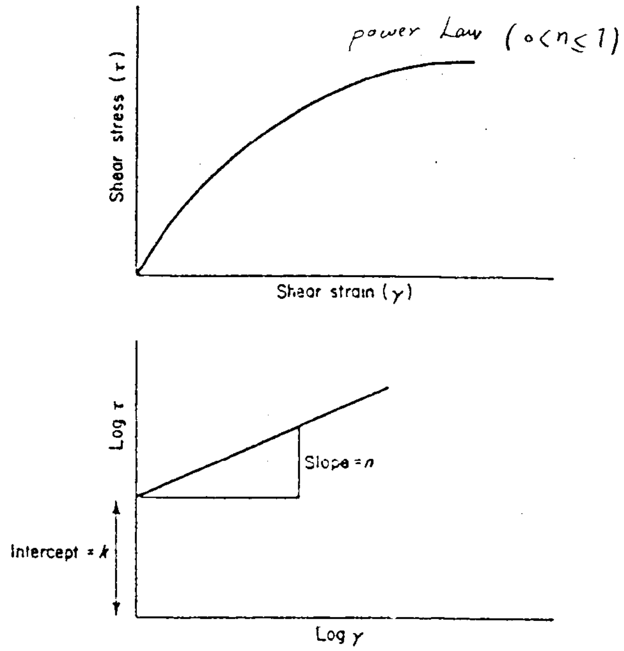

Fig. 5.7. Power-law fluid: (a) power-law relationship on a linear scale; (b) power-law relationship on a log–log scale.

(۳) Viscoplastic — τ تابعی از زمان است.

(۱) Thixotropic — کاهش τ با زمان (γ = cte)

(۲) Rheopectic — افزایش τ با زمان (γ, T = cte)

Time-dependent fluid

Bingham plastic and power-law fluids are referred to as time-independent fluids, in which the magnitude of viscosity is not affected by the duration of shear. A time-dependent fluid is one whose apparent viscosity at a fixed value of shear rate and temperature changes with the duration of shear. Two types of time-dependent fluids can be recognised:

(1) A *thixotropic fluid* exhibits a decrease in shear stress with duration of shear at constant shearing rate. A thixotropic fluid gels when it is static but returns to the liquid state upon agitation. Examples of thixotropic fluids include paint, grease and solutions of polymers.

(2) A *rheopectic fluid* exhibits an increase in shear stress with duration of shear at a given shearing rate and constant temperature. True rheopectic fluids are rare. Gypsum and bentonite suspensions are examples.

VISCOMETERS

The important rheological properties of fluids are normally measured with a viscometer (or a rheometer). The rotational type is typically designed to

(p.65)

rotate at two different speeds, namely 600 and 300 rpm, or at six speeds of 3, 6, 100, 200, 300 and 600 rpm. Most field instruments have two speeds only: 300 and 600 rpm.

The apparatus is so designed that the plastic viscosity is simply given by the difference between the shear stress at 600 rpm and 300 rpm. Thus,

$$PV = \theta_{600} - \theta_{300}$$

where θ_{600} = dial reading at 600 rpm; and θ_{300} = dial reading at 300 rpm.

From the Bingham plastic model,

$$\tau = YP + (PV)\gamma \tag{5.6}$$

At 300 rpm, $\tau = \theta_{300}$ and $\gamma = \gamma_{300}$. Therefore, Equation (5.6) becomes

$$\theta_{300} = YP + (PV)\gamma_{300}$$

or

$$(PV)\gamma_{300} = \theta_{300} - YP \tag{5.6a}$$

Similarly, at 600 rpm, Equation (5.6) becomes

$$(PV)\gamma_{600} = \theta_{600} - YP \tag{5.6b}$$

and

$$(PV)2\gamma_{300} = \theta_{600} - YP \tag{5.6c}$$

where $2\gamma_{300} = \gamma_{600}$. Dividing Equation (5.6c) by Equation (5.6a) yields

$$2 = \frac{\theta_{600} - YP}{\theta_{300} - YP}$$

or

$$YP = 2\theta_{300} - \theta_{600}$$
$$YP = \theta_{300} - PV \tag{5.8}$$

(*Note:* $PV = \theta_{600} - \theta_{300}$.)

DERIVATION OF LAMINAR FLOW EQUATIONS

$\tau = f(\gamma)$ only

Bingham plastic model

Fluid flow equations for pipes are normally derived using the following assumptions:

(1) Fluid velocity at the pipe wall is zero. This assumption effectively means that there is no slippage at the pipe wall.

(2) The magnitude of viscosity is independent of time or duration of shear. In other words, the fluid considered is a time-independent one and shear stress is a function of shear strain only.

(3) Particles moving within a cylindrical shell of a finite thickness travel parallel to the pipe axis with the same velocity. Particles contained within the shell adjacent to the pipe wall will have zero velocity. The velocity of particles in adjacent shells increases progressively towards the centre, until a maximum value is attained by particles contained within the central shell.

Pipe flow

Consider a concentric cylindrical shell of radius r and length L, as shown in Figure 5.8. During steady state flow of fluid, i.e. when the fluid is not accelerating, the following forces act on the shell: (a) differential pressure, $(P_1 - P_2)$, which causes the fluid to move with a steady speed of V; and (b) shear stress, resulting from the sliding action of particles within the shell over particles immediately outside this surface. The shear stress, τ, opposes the forward motion of the fluid particles within the shell, and for steady state flow an equilibrium between the forces exists such that

$$\text{forward force} = \text{opposing force}$$

$$(P_1 - P_2)\,\text{end area} = \tau(\text{surface area of shell})$$

But

$$\text{surface area of shell} = (2\pi r)L$$

Therefore,

$$(P_1 - P_2)\pi r^2 = 2\pi r L \times \tau$$

or

$$\boxed{\tau = \frac{\Delta p r}{2L}} \tag{5.9}$$

where Δp is the frictional pressure loss $(P_1 - P_2)$ or the pressure drop.

Distribution of shear stress Equation (5.9) can be used to determine the shear stress distribution during fluid flow. At the pipe wall

$$r = D/2 \text{ and } \tau = \tau_w$$

where τ_w is the pipe wall shear stress. By use of these values, Equation (5.9) becomes

$$\tau_w = \frac{\Delta p D/2}{2L}$$

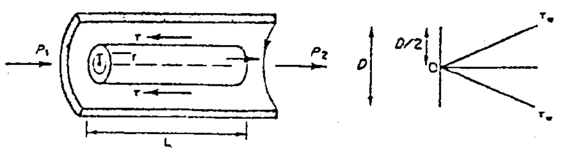

Fig. 5.8. Pipe flow.

(p.66)

or

$$\tau_w = \frac{\Delta p D}{4L} \tag{5.9a}$$

From Equations (5.9) and (5.9a) we obtain shear stress at any point in terms of τ_w and r as follows:

$$\tau = \frac{2r}{D}\tau_w \tag{5.9b}$$

where r is the radial distance from the centre of the pipe. Hence, from Equation (5.9b),

$$\text{at } r = 0,\ \tau = 0$$

The shear stress distribution is shown in Figure 5.8.

Laminar pipe flow equation The laminar fluid flow equation is developed by writing the Bingham plastic model, substituting Equation (5.9) into this model and integrating the resulting equation.

The Bingham plastic model states that

$$\checkmark\ \tau = \mathrm{YP} + (\mathrm{PV})\left(-\frac{dv}{dr}\right) \tag{5.6}$$

Substitution of Equation (5.9) in Equation (5.6) gives

$$\checkmark\ \frac{\Delta pr}{2L} = \mathrm{YP} + (\mathrm{PV})\left(-\frac{dv}{dr}\right)$$

Integrating with respect to r yields

$$\int \frac{\Delta pr}{2L}\,dr = \int \mathrm{YP}\,dr - \int (\mathrm{PV})\frac{dv}{dr}\,dr$$

$$\frac{\Delta p}{2L}\frac{r^2}{2} = \mathrm{YP}r - \mathrm{PV}\cdot V + C \tag{5.10}$$

At the pipe boundary

$$V = 0,\ r = R$$

where R is the radius of the pipe. Therefore,

$$C = \frac{\Delta p R^2}{4L} - \mathrm{YP}\cdot R \tag{5.10a}$$

Substituting for C in Equation (5.10) yields

$$\frac{\Delta p}{4L}r^2 = \mathrm{YP}\cdot r - (\mathrm{PV})V + \left(\frac{\Delta p R^2}{4L} - \mathrm{YP}R\right)$$

or

$$V = \frac{\Delta p}{4L(\mathrm{PV})}(R^2 - r^2) + \frac{\mathrm{YP}}{\mathrm{PV}}(r - R) \tag{5.10b}$$

Since it is more convenient to express flow equations in terms of volume flow rate, Q, rather than velocity, Equation (5.10b) is further simplified.

Using

$$dQ = V\,dA = V\,2\pi\,r dr$$

and the value of V from Equation (5.10b), we obtain

$$Q = \int_0^R V\,2\pi\,r dr$$

$$= 2\pi \int_0^R \left\{\frac{\Delta p}{4L(\mathrm{PV})}(R^2 - r^2) + \frac{\mathrm{YP}}{\mathrm{PV}}(r - R)\right\} r dr$$

$$= 2\pi \left[\frac{\Delta p}{4L(\mathrm{PV})}\left(R^2\frac{r^2}{2} - \frac{r^4}{4}\right) + \frac{\mathrm{YP}}{\mathrm{PV}}\left(\frac{r^3}{3} - \frac{r^2 R}{2}\right)\right]_0^R$$

$$= 2\pi \left[\frac{\Delta p}{4L(\mathrm{PV})}\left(\frac{R^4}{2} - \frac{R^4}{4}\right) + \frac{\mathrm{YP}}{\mathrm{PV}}\left(\frac{R^3}{3} - \frac{R^3}{2}\right)\right]$$

$$= \frac{\pi \Delta p}{8L(\mathrm{PV})}R^4 - \frac{\pi(\mathrm{YP})}{3(\mathrm{PV})}R^3$$

(N.B. The reader should note that the above result is only an approximate solution. The correct result is obtained by splitting the integral into two regions: (1) $\tau < \mathrm{YP}$, and (2) $\tau > \mathrm{YP}$. The final result should include a $(\mathrm{YP})^4$ term. In practice, the approximate solution was found to provide acceptable results.)

But average velocity

$$\bar{V} = \frac{Q}{A} = \frac{\dfrac{\pi \Delta p R^4}{8L(\mathrm{PV})} - \dfrac{\pi(\mathrm{YP})R^3}{3(\mathrm{PV})}}{\pi R^2}$$

$$= \frac{\Delta p R^2}{8L(\mathrm{PV})} - \frac{\mathrm{YP}R}{3(\mathrm{PV})}$$

Rearranging,

$$\Delta p = \frac{8L(\mathrm{PV})\bar{V}}{R^2} + \frac{8L(\mathrm{YP})}{3R} \tag{5.11}$$

Equation (5.11) is in consistent units and must be converted to oil field units. The conversion is carried out as follows.

✓*Imperial units*

First term

$$= \frac{8L(\mathrm{PV})\bar{V}}{R^2}$$

$$= \frac{8(\mathrm{ft})\left(\mathrm{cP} \times \dfrac{2.0886 \times 10^{-5}\ \mathrm{lb\ s}}{\mathrm{cP}\qquad \mathrm{ft}^2}\right)\left(\dfrac{\mathrm{ft}}{\mathrm{min}} \times \dfrac{\mathrm{min}}{60\ \mathrm{s}}\right)}{\mathrm{in}^2}$$

$$= \frac{16.7088 \times 10^{-5}}{60}\left(\frac{\mathrm{lb}}{\mathrm{in}^2}\right) = \frac{1}{359\,092}\ \mathrm{psi}$$

$$\text{Second term} = \frac{8L(\mathrm{YP})}{3R} = \frac{8(\mathrm{ft})\left(\dfrac{\mathrm{lb}}{100\ \mathrm{ft}^2} \times \dfrac{\mathrm{ft}^2}{144\ \mathrm{in}^2}\right)}{3\ \mathrm{in}\ \dfrac{\mathrm{ft}}{12\ \mathrm{in}}}$$

$$= \frac{1}{450}\ \mathrm{psi}$$

(P. 67)

Hence, Equation (5.11) becomes

$V = \ell t/\text{min}$

$$\Delta p \simeq \frac{L(\text{PV})\bar{V}}{90\,000D^2} + \frac{L(\text{YP})}{225D} \text{ psi} \quad (5.12)$$

where $D = 2R$.

Metric units

$$\text{First term} = \frac{8L(\text{PV})\bar{V}}{R^2} = \frac{8(\text{m})(\text{cP})\text{ m/s}}{\text{mm}^2 \dfrac{1\text{ m}^2}{10^6\text{ mm}^2}}$$

But

1 poise = 10^{-1} Pa.s

$$1\text{ cP} = 10^{-3}\frac{\text{kg}}{\text{m s}}$$

Therefore,

$$\text{first term} = \frac{8(\text{m})\left(\text{cP} \times \dfrac{10^{-3}\text{ kg/m s}}{\text{cP}}\right)\text{m/s}}{\text{mm}^2(1\text{ m}^2/10^6\text{ mm}^2)}$$

$$= 8000\text{ N/m}^2$$

$$\text{Second term} = \frac{8L(\text{YP})}{3R} = \frac{8(\text{m})(0.479\text{ N/m}^2)}{3\text{ mm} \times \dfrac{1\text{ m}}{10^3\text{ mm}}}$$

It should be noted that the yield point of a drilling mud is measured in the field using a viscometer. The instrument gives the value of yield point directly in lb/100 ft^2 calculated as the difference between twice the reading at 300 rpm and the reading at 600 rpm (see Chapter 6). As there is no metric version of the viscometer to date, yield point is still measured in lb/100 ft^2 and later converted to N/m^2. The conversion factor from lb/100 ft^2 to N/m^2 is 0.479.

Therefore, the constant of the second term

$$= \frac{8 \times 0.479}{3 \times 10^3} = 1.277 \times 10^3\text{ N/m}^2$$

Hence, Equation (5.11) becomes:

$$\Delta p = \left(\frac{8000L(\text{PV})\bar{V}}{R^2}\right) + \frac{1277L(\text{YP})}{R}\text{ N/m}^2$$

Since diameter $D = 2R$, the above equation simplifies to

$$\Delta p = \frac{32\,000L(\text{PV})\bar{V}}{D^2} + \frac{2554L(\text{YP})}{D}\text{ N/m}^2 \quad (5.13)$$

Pressure is normally expressed in bar (or kPa), and the above equation becomes

$$\Delta p = \frac{0.32L(\text{PV})\bar{V}}{D^2} + \frac{0.025\,54L(\text{YP})}{D}\text{ bar} \quad (5.13\text{a})$$

or

$$\Delta p = \frac{32L(\text{PV})\bar{V}}{D^2} + \frac{2.55L(\text{YP})}{D}\text{ kPa} \quad (5.13\text{b})$$

Critical velocity in pipe flow

Equations (5.12) and (5.13) are only applicable to laminar flow. In the previous sections it was indicated that laminar flow occurs at $(Re) \leq 2000$ and turbulent flow at $(Re) > 3000$. Hence, an expression for the critical velocity can be obtained by letting $(Re) = 3000$.

As discussed previously, a Newtonian fluid is one in which YP = 0. Thus, substituting YP = 0 in Equation (5.13) gives

$$\Delta p = \frac{32\,000\,L\mu_e\,\bar{V}}{D^2}\text{ N/m}^2 \quad (5.14)$$

where μ_e is the effective viscosity. Equation (5.14) is also known as the Hagen–Poiseuille equation.

A value for effective viscosity (μ_e) may be used in Equation (5.14) so that the magnitude of Equation (5.14) is numerically equal to that of Equation (5.13). Thus,

$$\frac{32\,000L\mu_e V}{D^2} = \frac{3200L(\text{PV})V}{D^2} + \frac{2554LYP}{D}$$

Therefore,

$$\mu_e = \text{PV} + \frac{2554}{32\,000}\left(\frac{D}{V}\right)\text{YP} \quad (5.15)$$

Also (for metric units),

$$(Re) = \frac{1000\bar{V}D\rho}{\mu_e} \quad (5.15\text{a})$$

And for turbulent flow $Re = 3000$. Hence, Equation (5.15a) becomes

$$3000 = \frac{1000V_cD\rho}{\mu_e} \quad (5.16)$$

where V_c is the critical velocity at which turbulence takes place.

Substituting μ_e from Equation (5.15) in Equation (5.16), we obtain

$$3000 = \frac{\rho V_c D}{\mu_e} = \frac{\rho V_c D}{(\text{PV}) + \dfrac{2554}{32\,000}\dfrac{(D\text{YP})}{V_c}}$$

Simplifying,

$$\tfrac{1}{3}\rho D\,V_c^2 - (\text{PV})V_c - 0.0798\,D\text{YP} = 0$$

(p. 68)

Since velocity cannot be negative, only the positive root of the above equation is useful. Therefore,

$$V_c = \frac{PV + \sqrt{PV^2 + 4\left(\frac{\rho D}{3}\right)(0.0798\, D YP)}}{\frac{2}{3}\rho D}$$

$$= 1.5\left(\frac{PV + \sqrt{PV^2 + 0.1064\rho D^2 YP}}{\rho D}\right) \text{m/s} \tag{5.17}$$

Hence, Equation (5.17) can be used as a criterion for determining whether the flow is laminar or turbulent by simply comparing velocity V_c with the average velocity of fluid as determined from $\bar{V} = Q/A$.

If $V_c < \bar{V}$, flow is turbulent, and if $V_c > \bar{V}$, flow is laminar and Equation (5.12) or Equation (5.13) is used to determine pressure drop.

In order to obtain V_c in Imperial units, the same procedure as outlined above is used to simplify Equation (5.12) to obtain

$$\mu_e = PV + 400\ YP\left(\frac{D}{V}\right) \tag{5.18}$$

and, for turbulent flow,

✓(Critical Re) $(Re) = 3000 = 15.46\dfrac{\rho V D}{\mu_e}$ (5.19)

From Equations (5.18) and (5.19), we obtain

$$V_c = \frac{97\ PV + 97\sqrt{PV^2 + 8.2\ \rho D^2\ YP}}{\rho D} \text{ ft/min} \tag{5.20}$$

It should be observed that other forms of Equation (5.20) have appeared in the literature in which turbulence was assumed to start at Reynolds numbers of 2000 or 2500, viz.

$(Re) = 2000$

$$V_c = \frac{1.08\ PV + 1.08\sqrt{PV^2 + 12.3\ \rho D\ YP}}{\rho D} \text{ ft/s} \tag{5.20a}$$

$(Re) = 2500$

$$V_c = \frac{1.13\ PV + 1.13\sqrt{PV^2 + 8.8\ \rho D\ YP}}{\rho D} \text{ ft/s} \tag{5.20b}$$

✓ Example 5.1

A drilling mud is pumped at the rate of 200 gal/min through a drillpipe of 4.5 in internal diameter and 400 ft in length. The fluid has a density of 9 lbm/gal, a plastic viscosity of 15 cP and a yield point of 10 lb/100 ft². Determine the type of flow and the magnitude of the pressure drop through the drillpipe.

Solution

Using Equation (5.20), determine the critical velocity of the fluid, V_c:

$$V_c = \frac{97 \times 15 + 97\sqrt{(15)^2 + 8.2 \times 9 \times (4.5)^2 \times 10}}{9 \times 4.5}$$

$$= 330.9 \text{ ft/min}$$

$$\text{Average velocity } \bar{V} = \frac{Q}{A}$$

or

$$\bar{V} = \frac{24.5Q}{D^2} \text{ ft/min}$$

$$\bar{V} = \frac{24.5 \times 200}{(4.5)^2} = 242 \text{ ft/min}$$

Since $V_c > \bar{V}$, flow is laminar, and Equation (5.12) is applicable:

$$\Delta p = \frac{L(PV) \times \bar{V}}{90\ 000 D^2} + \frac{L\ YP}{225 D}$$

$$= \frac{400 \times 15 \times 241.9}{90\ 000 \times (4.5)^2} + \frac{400 \times 10}{225 \times (4.5)}$$

$$= 0.80 + 3.95$$

$$\Delta p = 4.8 \text{ psi}$$

Annular flow

Several versions of annular equations for laminar flow have appeared in the literature, each producing a different value of pressure drop for the same volume flow rate and fluid properties. In oil well drilling, laminar annular pressure losses are small, representing less than 10% of total pressure losses, and are normally incorporated within the turbulent losses[1]. In oil well drilling, annular flow is encountered when mud flows through the annulus between the drillpipe (or drill collars) and casing (or hole).

Without significant loss of accuracy, the annular geometry may be represented by a narrow slot between two infinite plates, as shown in Figure 5.9. Under steady state flows, the forces acting on the narrow slot

$$\Delta p\ W \times E = 2\tau_w (L \times E)$$

$$\tau_w = \frac{\Delta p\ W}{2L}$$

(p.69)

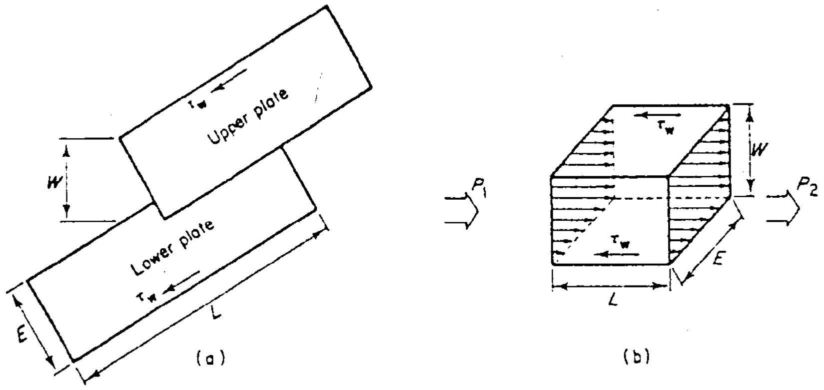

Fig. 5.9. Fluid in a narrow slot: (a) flow in a slot; (b) cubic element within the slot. W = vertical distance separating plates; L = length of plate (infinite); E = width of plate.

where $\Delta p = P_1 - P_2$; τ_w = shear stress at the pipe wall; E = width of plate; L = length of plate; and W = vertical distance between plates.

If the distance between the centre of this cube and the extreme boundary is taken as Y, then

$$W = 2Y$$

Hence,

$$\tau_w = \frac{\Delta p\, 2Y}{2L} = \frac{\Delta p\, Y}{L}$$

At any distance, y, from the centre, this equation may be modified to give shear stress, as follows:

$$\boxed{\tau = \frac{\Delta p \cdot y}{L}} \tag{5.21}$$

The Bingham plastic model states that

shear stress = yield stress + plastic viscosity × rate of shear

or

$$\tau = \text{YP} + \text{PV}\left(\frac{-\mathrm{d}v}{\mathrm{d}y}\right) \tag{5.6}$$

Substituting Equation (5.21) into Equation (5.6) yields

$$\frac{\Delta p y}{L} = \text{YP} + (\text{PV})\left(\frac{-\mathrm{d}v}{\mathrm{d}y}\right)$$

Integrating with respect to y,

$$\int \frac{\Delta p}{L} y \,\mathrm{d}y = \int (\text{YP})\,\mathrm{d}y - \int (\text{PV})\,\mathrm{d}v$$

$$\left(\frac{\Delta p}{2L}\right) y^2 = \text{YP}\, y - (\text{PV})v + C \tag{5.22}$$

where C is a constant. At the boundary $y = Y$ and $v = 0$. Therefore,

$$C = \left(\frac{\Delta p}{2L}\right) Y^2 - \text{YP}\, Y \tag{5.22a}$$

Substituting Equation (5.22a) in Equation (5.22) and rearranging yields

$$v = \frac{\Delta p}{2L(\text{PV})}(Y^2 - y^2) + \frac{\text{YP}}{\text{PV}}(y - Y) \tag{5.22b}$$

But

$$\mathrm{d}Q = \text{volume flow rate} = v \cdot \mathrm{d}A$$

where A = slot area = WE = $E(2Y)$ (see Figure 5.9); $\mathrm{d}A = E(2 \cdot \mathrm{d}y)$. Substituting the value of v from Equation (5.22b) in the expression for $\mathrm{d}Q$ we obtain

$$Q = \int_0^Y \left\{\frac{\Delta p}{2L(\text{PV})}(Y^2 - y^2) + \frac{\text{YP}}{(\text{PV})}(y - Y)\right\} E(2 \cdot \mathrm{d}y)$$

$$= \left[\frac{\Delta p}{2L(\text{PV})}(Y^2 y - y^3/3) + \frac{\text{YP}}{\text{PV}}\left(\frac{y^2}{2} - yY\right)\right]_0^Y 2E$$

$$= \frac{\Delta p}{2L(\text{PV})}\left(Y^3 - \frac{Y^3}{3}\right) + \frac{\text{YP}}{\text{PV}}\left(\frac{Y^2}{2} - Y^2\right) 2E$$

$$= \left[\frac{\Delta p}{L(\text{PV})}\left(\frac{2}{3}Y^3\right) - \frac{\text{YP}}{\text{PV}} Y^2\right] E$$

and average velocity is given by

$$\bar{V} = \frac{Q}{A}$$

$$\bar{V} = \frac{\left(\dfrac{2\Delta p}{3L(\text{PV})} Y^3 - \dfrac{\text{YP}}{\text{PV}} Y^2\right)}{2YE} \cdot E$$

(P. 70)

Therefore,

$$\bar{V} = \frac{\Delta p Y^2}{3L(\text{PV})} - \frac{\text{YP}Y}{2(\text{PV})}$$

Solving for Δp and using $Y = \frac{W}{2} = \frac{1}{2}\left(\frac{D_h - D_p}{2}\right) = \left(\frac{D_h - D_p}{4}\right)$

where D_h = hole diameter; D_p = drillpipe (or collar) outside diameter yields

$$\Delta p = \frac{48L(\text{PV})\bar{V}}{(D_h - D_p)^2} + \frac{6L(\text{YP})}{(D_h - D_p)} \quad (5.23)$$

Imperial units Equation (5.23) can be converted to field units (in order to express Δp in units of psi) by the following method.

$$\text{First term} = \frac{48L(\text{PV})\bar{V}}{(D_h - D_p)^2}$$

$$= \frac{48\ (\text{ft})}{\text{in}^2}\left(\text{cP} \times \frac{2.0886 \times 10^{-5}\ \text{lb s}}{\text{cP ft}^2}\right) \times \left(\frac{\text{ft}}{\text{min}} \times \frac{\text{min}}{60\ \text{s}}\right)$$

$$\text{Constant of first term} = \frac{48 \times 2.0886}{60 \times 10^5} = \frac{1}{59\,848.7}\frac{\text{lb}}{\text{in}^2} \simeq \frac{1}{60\,000}\ \text{psi}$$

$$\text{Second term} = \frac{6L(\text{YP})}{(D_h - D_p)} = \frac{6 \times \text{ft}}{\text{in}}\left(\frac{\text{lb}}{100\ \text{ft}^2}\right)$$

$$= \frac{6\left(\text{ft} \times \frac{12\ \text{in}}{\text{ft}}\right)}{100 \times \text{in}}(\text{lb})\left(\frac{1}{\text{ft}^2} \times \frac{\text{ft}^2}{144\ \text{in}^2}\right)$$

$$= \frac{6 \times 12}{14\,400}\frac{\text{lb}}{\text{in}^2} = \frac{1}{200}\ \text{psi}$$

Hence, Equation (5.23) becomes, in field units,

$$\Delta p = \frac{L(\text{PV})\bar{V}}{60\,000(D_h - D_p)^2} + \frac{L(\text{YP})}{200(D_h - D_p)}\ \text{psi} \quad (5.24)$$

Metric units

$$\text{First term} = \frac{48\ \text{m}\left(\text{cP} \times \frac{10^{-3}\ \frac{\text{kg}}{\text{ms}}}{\text{cP}}\right)(\text{m/s})}{\text{mm}^2 \times \frac{\text{m}^2}{10^6\ \text{mm}^2}}$$

$$= 48\,000\ \text{N/m}^2$$

$$\text{Second term} = \frac{6\ \text{m} \times (0.479\ \text{N/m}^2)}{\text{mm} \times \frac{\text{m}}{10^3\ \text{mm}}}$$

$$= 2874\ \text{N/m}^2$$

Hence, Equation (5.23) becomes

$$\Delta p = \frac{48000L(\text{PV})\bar{V}}{(D_h - D_p)^2} + \frac{2874L(\text{YP})}{(D_h - D_p)}\ \text{N/m}^2 \quad (5.25)$$

metric unit

or

$$\Delta p = \frac{48L(\text{PV})\bar{V}}{(D_h - D_p)^2} + \frac{2.874L(\text{YP})}{(D_h - D_p)}\ \text{kPa} \quad (5.26)$$

or

$$\Delta p = \frac{0.48L(\text{PV})\bar{V}}{(D_h - D_p)^2} + \frac{0.0287L(\text{YP})}{(D_h - D_p)}\ \text{bar} \quad (5.27)$$

Example 5.2

Determine the annular pressure losses using the following data:

length of drill pipe	= 9000 ft
drill pipe	= OD/ID: 5 in/4.276 in
hole diameter	= 8.5 in
PV	= 20 cP
yield point	= 20 lb/100 ft^2
flow rate	= 300 gpm
mud density	= 10 ppg

Solution

Equation (5.24)

$$\Delta p = \frac{L(\text{PV})\bar{V}}{60\,000(D_h - D_p)^2} + \frac{L(\text{YP})}{200(D_h - D_p)}$$

$$\bar{V} = \frac{24.5Q}{(D_h^2 - D_p^2)} = \frac{24.5 \times 300}{(8.5)^2 - (5)^2} = 155.6\ \text{ft/min}$$

(*Note:* The critical velocity is 402 ft/min, indicating that the flow is laminar.)

$$\Delta p = \frac{9000 \times 20 \times 155.6}{60\,000(8.5 - 5)^2} + \frac{9000 \times 20}{200(8.5 - 5)}$$

$$= 38.1 + 257.1 = 295.2\ \text{psi}$$

Critical velocity in annular flow

Using the same argument as given above (page 92), Equation (5.25) can be used to describe Newtonian

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fluid by letting YP = 0. Therefore,

$$\Delta p = \frac{48\,000\, L\mu_e v}{(D_h - D_p)^2} \tag{5.28}$$

where μ_e is the effective viscosity of the fluid.

A value of μ_e may be used in Equation (5.28) so that this equation is numerically equal to Equation (5.25). Hence,

$$\frac{48\,000\, L\mu_e\bar{v}}{(D_h - D_p)^2} = \frac{48\,000\, L\,(\mathrm{PV})\,\bar{v}}{(D_h - D_p)^2} + \frac{2874\, L\,(\mathrm{YP})}{(D_h - D_p)^2}$$

$$\mu_e = \mathrm{PV} + \frac{2874}{48\,000}\,\frac{D_e\,\mathrm{YP}}{\bar{v}} \tag{5.28b}$$

where $D_e = d_h - D_p$.

By use of $(Re) = \dfrac{1000\, vD_e}{\mu_e}$, $(Re) = 3000$ for turbulent flow and replacing $\bar{v}$ by V_c, Equation (5.28b) becomes

$$3000 = 1000\frac{\rho V_c D_e}{\left[\mathrm{PV} + \dfrac{2874}{48\,000}\,\dfrac{D_e(\mathrm{YP})}{V_c}\right]}$$

Taking positive roots only, we obtain:

$$V_c = \frac{\mathrm{PV} + \sqrt{\mathrm{PV}^2 + 4\dfrac{\rho D_e}{3}(0.059)D_e(\mathrm{YP})}}{2\rho D_e/3}$$

$$V_c = 1.5\left[\frac{\mathrm{PV} + \sqrt{\mathrm{PV}^2 + 0.079\rho D_e^2(\mathrm{YP})}}{\rho D_e}\right]\ \mathrm{m/s} \tag{5.29}$$

In Imperial units critical velocity may be determined by using the above procedure to arrive at

$$V_c = \frac{97\,\mathrm{PV} + 97\sqrt{\mathrm{PV}^2 + 6.2\,\rho D_e^2(\mathrm{YP})}}{\rho D_e}\ \mathrm{ft/min} \tag{5.30}$$

$$\left[\textit{Note: } \mu_e = \mathrm{PV} + 300\,\mathrm{YP}\frac{(D_h - D_p)}{\bar{v}}\right]$$

Power-law model

The power-law model states that

$$\tau = K(\gamma)^n$$

or (5.31)

$$\log\tau = \log K + n\log\gamma$$

Hence, a graph of log τ against log γ is a straight line with a slope n and intercept of log K. Such a graph can be constructed by plotting the values of shear stress measured by a viscometer at 300 rpm and 600 rpm, as shown in Figure 5.10.

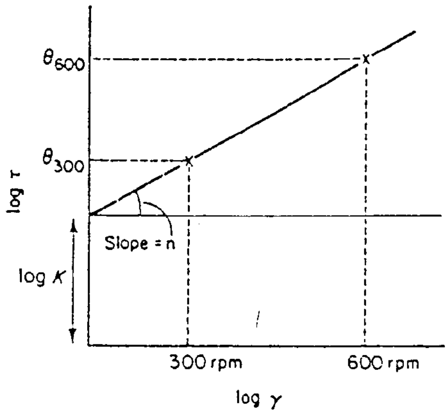

Fig. 5.10. Viscometer readings plotted on a log τ–log γ graph.

Hence,

$$\text{slope } (n) = \frac{\log\theta_{300} - \log K}{\log\gamma_{300}}$$

The viscometer is so designed that the shear strain at 300 rpm is equivalent to 511 s^{-1}. Therefore,

$$n = \frac{\log\theta_{300} - \log K}{\log 511}$$

$$K = \frac{\theta_{300}}{(511)^n} \tag{5.32}$$

Also,

$$n = \frac{\log\theta_{600} - \log\theta_{300}}{\log\gamma_{600} - \log\gamma_{300}}$$

where $\gamma_{600} = 1022\ \mathrm{s}^{-1}$

$$n = \frac{\log\left(\dfrac{\theta_{600}}{\theta_{300}}\right)}{\log 1022 - \log 511} = \frac{\log\left(\dfrac{\theta_{600}}{\theta_{300}}\right)}{\log\left(\dfrac{1022}{511}\right)}$$

or

$$n = 3.32\log\left(\frac{\theta_{600}}{\theta_{300}}\right) \tag{5.33}$$

Pipe flow

The power-law model equation can be rewritten as

$$\tau = K\left(\frac{-dv}{dr}\right)^n \tag{5.34}$$

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Shear stress, τ, in terms of Δp, r and L is also given by

$$\tau = \frac{\Delta p r}{2L} \tag{5.9}$$

Substituting Equation (5.9) in Equation (5.34) yields

$$\left(\frac{\Delta p r}{2L}\right) = K\left(\frac{-dv}{dr}\right)^n$$

$$\left(\frac{\Delta p}{2LK} r\right)^{1/n} = \left(\frac{-dv}{dr}\right)$$

$$\left(\frac{\Delta p}{2LK}\right)^{1/n} (r)^{1/n}\, dr = -dv$$

Integrating with respect to r gives

$$\left(\frac{\Delta p}{2LK}\right)^{1/n} \frac{r^{\left(\frac{n+1}{n}\right)}}{\left(\frac{n+1}{n}\right)} = -v + C \tag{5.34a}$$

where C is a constant.

At boundary $r = R$ and $v = 0$

$$C = \left(\frac{\Delta p}{2LK}\right)^{1/n} \times \frac{R^{[(n+1)/n]}}{\frac{n+1}{n}}$$

Hence, Equation (5.34a) becomes

$$\left(\frac{\Delta p}{2LK}\right)^{1/n} \frac{r^{[(n+1)/n]}}{\frac{n+1}{n}} = -v + \left(\frac{\Delta p}{2LK}\right)^{1/n} \frac{R^{[(n+1)/n]}}{\frac{n+1}{n}}$$

or

$$v = \left(\frac{\Delta p}{2LK}\right)^{1/n} \frac{n}{n+1} \left(R^{[(n+1)/n]} - r^{[(n+1)/n]}\right)$$

Using

$$Q = v\,dA = \int_0^R \left\{\left(\frac{\Delta p}{2LK}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{[(n+1)/n]} - r^{[(n+1)/n]}\right)\right\} 2\pi r\,dr$$

$$Q = \left(\frac{\Delta p}{2LK}\right)^{1/n}\left(\frac{n}{n+1}\right)\left[R^{[(n+1)/n]}\frac{r^2}{2} - \frac{r^{[(3n+1)/n]}}{\left(\frac{3n+1}{n}\right)}\right]_0^R 2\pi$$

$$\left[\textit{Note: } \int r^{[(n+1)/n]} r\, dr = r^{[(2n+1)/n]} \cdot dr = \frac{r^{[(2n+1)/n+1]}}{\left(\frac{2n+1}{n}+1\right)} = \left(\frac{n}{3n+1}\right) r^{[(3n+1)/n]}\right]$$

$$Q = \left(\frac{\Delta p}{2LK}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(\frac{R^{[(n+1)/n]}R^2}{2} - \frac{n}{3n+1} R^{[(3n+1)/n]}\right) 2\pi$$

Average velocity, $\bar{V}$, is given by

$$\bar{V} = \frac{Q}{A} = \frac{\left(\frac{\Delta p}{2LK}\right)^{1/n}\left(\frac{n}{n+1}\right) \times \left\{\frac{R^{[(n+1)/n]}}{2}R^2 - \frac{n}{3n+1}R^{[(3n+1)/n]}\right\} 2\pi}{\pi R^2}$$

Rearranging gives

$$\Delta p = \frac{2LK}{R}\left[\left(\frac{3n+1}{n}\right)\frac{\bar{V}}{R}\right]^n$$

Pa   m   K(Pa.s^n)   m/s   m

But $R = D/2$, where D is the diameter of the pipe. Therefore, (OD)?

$$\Delta p = \frac{4LK}{D}\left[2\left(\frac{3n+1}{n}\right)\frac{\bar{V}}{D}\right]^n \tag{5.35}$$

Field units of Equation (5.35)

Imperial units

$$K = \frac{\theta_{300}}{(511)^n} = \left(\frac{\text{lb}}{100\ \text{ft}^2} \times \frac{\text{ft}^2}{144\ \text{in}^2}\right)(\text{s}^{+1})^n$$

$$K = \frac{1}{14\,400}\ \text{psi}\ (\text{s}^{+1})^n$$

Therefore,

$$\Delta p = \text{LK}\frac{4\ \text{ft} \times \frac{1}{14\,400}\text{psi s}^n}{\text{in} \times \frac{\text{ft}}{12\ \text{in}}} \times \left[2\left(\frac{3n+1}{n}\right)\frac{(\text{ft/min})\frac{\text{min}}{60\ \text{s}}\bar{V}}{\left(\text{in} \times \frac{\text{ft}}{12\ \text{in}}\right)\bar{D}}\right]^n$$

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D : in K : lbf/100 ft²
$\bar{V}$: ft/min L : ft

Δp : psi

$$\Delta p = \frac{1}{300D} KL \left[\frac{2}{5} \left(\frac{3n+1}{n} \right) \frac{\bar{v}}{D} \right]^n \quad (5.36)$$

where $\bar{V}$ is in ft/min and Δp is in psi.

Metric units In SI units the various terms of Equation (5.35) have the following units: $K = \mathrm{Pa} \cdot s^n$, $L = \mathrm{m}$, $D = \mathrm{m}$, $\bar{V} = \mathrm{m/s}$. Hence, Equation (5.35) gives the pressure drop in Pa (N/m²) directly.

In oil well drilling the units of length and diameter are given in metres and millimetres, respectively, while the consistency index K is determined from viscometer readings. The units of K as determined from this apparatus are in $\mathrm{lb} \cdot s^n/100\ \mathrm{ft}^2$ and must, therefore, be converted to $\mathrm{Ns}^n/\mathrm{m}^2$ or $\mathrm{Pa} \cdot s^n$. Hence, the power-law equations in this book will be modified to suit field requirements in order to produce the units of pressure in bars, N/m² or kPa.

Since

$$K = \frac{\theta_{300}}{(511)^n} \frac{\mathrm{lb}}{100\ \mathrm{ft}^2}\, s^n$$

where θ_{300} is measured in lb/100 ft²; therefore, to convert the value of K to $\mathrm{N/m^2} \cdot s^n$ multiply by a factor of 0.479.

Hence the metric field version of Equation (35) is:

$$\Delta p = \frac{4\ \mathrm{m} \times 0.479 \frac{\mathrm{N}}{\mathrm{m}^2} s^n}{\mathrm{mm} \times \frac{\mathrm{m}}{10^3\ \mathrm{mm}}} \frac{LK}{D}$$

$$\times \left[2\left(\frac{3n+1}{n} \right) \mathrm{mm} \times \frac{\mathrm{m/s}}{10^3\ \mathrm{mm}} \right]^n$$

Δp : Pa

$$\Delta p = \frac{1916KL}{D} \left[2000 \left(\frac{3n+1}{n} \right) \frac{V}{D} \right]^n \mathrm{N/m^2} \quad (5.37)$$

$$\Delta p = 19.16 \times 10^{-3} \frac{KL}{D} \left\{ 2000 \left(\frac{3n+1}{n} \right) \frac{\bar{V}}{D} \right\}^n \mathrm{bar} \quad (5.37a)$$

In Equations (5.37) and (5.37a) K is measured in lb s^n/100 ft².

Annular flow

Equation (21) states that

$$\tau = \frac{\Delta p y}{L}$$

Substituting Equation (5.21) in Equation (5.31), we obtain

$$\tau = \frac{\Delta p y}{L} = K(\gamma)^n = K\left(\frac{-\mathrm{d}v}{\mathrm{d}y} \right)$$

where $\gamma = (-\mathrm{d}v)/\mathrm{d}y$. Therefore,

$$\left(\frac{\Delta p}{LK} \right)^{1/n} y^{1/n} = \frac{-\mathrm{d}v}{\mathrm{d}y}$$

Integrating with respect to y gives

$$\left(\frac{\Delta p}{LK} \right)^{1/n} \frac{y^{[(n+1)/n]}}{\frac{n+1}{n}} = v + C$$

where C is a constant.

Boundary conditions give $v = 0$ at $y = Y$.

$$C = \left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) Y^{[(n+1)/n]}$$

Therefore,

$$v = \left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) \{ Y^{[(n+1)/n]} - y^{[(n+1)/n]} \}$$

Using $Q = v\,\mathrm{d}A$ and the value of r from the above equation yields

$$Q = \int_0^Y \left\{ \left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) [Y^{[(n+1)/n]} - y^{[(n+1)/n]}] \right\} E(2 \cdot \mathrm{d}y)$$

where E = width of plate (see Figure 5.9).

$$Q = \left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) \left[Y^{[(n+1)/n]} y - \left(\frac{n}{2n+1} \right) y^{[(2n+1)/n]} \right]_0^Y 2E$$

$$= \left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) Y^{[(2n+1)/n]} \left[\frac{n+1}{(2n+1)} \right] 2E$$

Average velocity, $\bar{V}$, is given by

$$\bar{V} = \frac{Q}{A} = \frac{\left(\frac{\Delta p}{LK} \right)^{1/n} \left(\frac{n}{n+1} \right) y^{[(2n+1)/n]} \left(\frac{n+1}{2n+1} \right) 2E}{2YE}$$

or

$$\Delta p = \frac{KL}{Y} \left[\left(\frac{2n+1}{n} \right) \frac{\bar{V}}{Y} \right]^n$$

Referring to Figure 5.9,

$$Y = \frac{W}{2} = 1/2 \left(\frac{D_h - D_p}{2} \right) = \frac{1}{4}(D_h - D_p)$$

$$\Delta p = \frac{4KL}{D_h - D_p} \left[4 \frac{(2n+1)}{n} \frac{\bar{V}}{(D_h - D_p)} \right]^n \quad (5.38)$$

✓ Consistent Units

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Equation (5.38) is in consistent units and must be converted to field units.

Field units of Equation (5.38)

Imperial units Constant of Equation (5.38)

$$= \frac{4\left(\frac{1}{14\,400}\right)(\text{psi s}^n)(\text{ft})}{\text{in}\left(\frac{\text{ft}}{12\text{ in}}\right)}\left[4\frac{\text{ft}}{\text{min}}\times\frac{1\text{ min}}{60\text{ s}}\frac{1}{\text{in}\left(\frac{\text{ft}}{12\text{ in}}\right)}\right]^n$$

$$= \frac{1}{300}\text{ psi s}^n\left[\frac{48}{60}\frac{1}{\text{s}}\right]^n$$

Equation (5.38) becomes Field Units

$$\Delta p = \frac{KL}{300(D_h - D_p)}\left[\frac{4}{5}\left(\frac{2n+1}{n}\right)\frac{\bar{V}}{D_h - D_p}\right]^n \quad (5.39)$$

where Δp is in psi.

Metric units

$$\Delta p = \frac{4\times\text{m}\times 0.479\,(\text{N/m}^2)\,\text{s}^n}{\text{mm}\times\left(\frac{\text{m}}{1000\text{ mm}}\right)}$$

$$\times\left[4\left(\frac{2n+1}{n}\right)\frac{\text{m/s}}{\text{mm}\times\frac{\text{m}}{1000\text{ mm}}}\right]^n$$

$$\Delta p = 1916\times 10^{-3}\frac{KL}{(D_h - D_p)}$$ SI (metric) field

$$\times\left[\frac{4000}{D_h - D_p}\left(\frac{2n+1}{n}\right)\bar{V}\right]^n \text{N/m}^2$$

or

$$\Delta p = 19.16\times 10^{-3}\frac{KL}{D_h - D_p}\left[\frac{4000}{D_h - D_p}\left(\frac{2n+1}{n}\right)\bar{V}\right]^n \text{bar} \quad (5.40)$$

Critical velocities for power-law model

Pipe flow

Metric units In Equation (5.14) it was shown that for a Newtonian fluid (YP = 0), pressure loss, Δp, is given by

$$\Delta p = \frac{32\,000\,L\mu_e\bar{V}}{D^2}\text{ N/m}^2$$

An effective viscosity, μ_e, may be substituted in Equation (5.14) such that Equation (5.14) is numerically equal to Equation (5.37), i.e.

$$\left(\frac{32\,000\,Lv}{D^2}\right)\mu_e = 1916\frac{KL}{D}\left[2000\left(\frac{3n+1}{n}\right)\frac{v}{D}\right]^n$$

$$\mu_e = \frac{1916}{32\,000}\frac{DK}{v}\left[2000\left(\frac{3n+1}{n}\right)\frac{v}{D}\right]^n \quad (5.41)$$

The equation for Reynolds Number in metric units is given by

$$(Re) = 1000\frac{DV\rho}{\mu_e} \quad (5.41a)$$

and when $(Re) = 3000$ flow is turbulent and $v = V_c$ (critical velocity), hence, substituting Equation (5.41) in Equation (5.41a), we obtain

$$3000 = 1000\frac{DV_c\rho}{\left\{\frac{1916}{32\,000}\frac{DK}{V_c}\left[2000\left(\frac{3n+1}{n}\right)\frac{V_c}{D}\right]^n\right\}}$$

$$= \frac{DV_c\rho}{0.179\frac{DK}{V_c}\left[2000\left(\frac{3n+1}{n}\right)\frac{V_c}{D}\right]^n}$$

$$V_c = \left[\frac{0.179K}{\rho}\right]^{[1/(2-n)]}\times\left[\frac{2000}{D}\left(\frac{3n+1}{n}\right)\right]^{[n/(2-n)]}\text{m/s} \quad (5.42)$$

Imperial units By the same procedures as for metric units, critical velocity in Imperial units may be shown to be

$$V_c = \left[\frac{5.82\times 10^4 K}{\rho}\right]^{[1/(2-n)]}\times\left[\frac{1.6}{D}\left(\frac{3n+1}{4n}\right)\right]^{[n/(2-n)]}\text{ft/min} \quad (5.43)$$

Annular flow

From equations (39) and (28) and the Reynolds Number equation, the critical velocities for annular flow can be determined as detailed for pipe flow:

Metric system

$$V_c = \left[\frac{0.119K}{\rho}\right]^{[1/(2-n)]}\times\left[\frac{4000}{D_h - D_p}\left(\frac{2n+1}{n}\right)\right]^{[n/(2-n)]}\text{m/s} \quad (5.44)$$

Imperial units

$$V_c = \left[\frac{3.878(10^4)K}{\rho}\right]^{[1/(2-n)]}\times\left[\frac{2.4}{D_h - D_p}\left(\frac{2n+1}{3n}\right)\right]^{[n/(2-n)]}\text{ft/min} \quad (5.45)$$

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The derivation of Equations (5.44) and (5.45) is left as an exercise for the reader.

TURBULENT FLOW

In the previous sections analytical techniques were used to describe laminar fluid flow and to derive expressions for pressure losses in pipes and annuli. Owing to the chaotic nature of fluid particle movement in turbulent flow, it is extremely difficult to arrive at an exact analytical method for determining pressure losses. A large number of workers in this field have arrived at the conclusion that pressure losses in turbulent flow are best determined from charts or by using the following equation:

$$\Delta p = \frac{2fL\rho \bar{V}^2}{D} \tag{5.46}$$

The friction factor, *f*

Solution of Equation (5.46) becomes straightforward once the value of the friction factor, f, is determined. In laminar flow the basic equation relating Δp to L, μ_e, $\bar{V}$ and D is given by Equation (5.11), when $PV = \mu_e$ and $YP = 0$, as

$$\Delta p = \frac{32\, L\mu_e V}{D^2}\ \mathrm{N/m^2} \tag{5.47}$$

where $D = 2R$. Equating Equations (5.46) and (5.47) yields

$$\frac{2fL\rho \bar{V}^2}{D} = \frac{32L\mu_e \bar{V}}{D^2}$$

$$f = \frac{16\mu_e}{\rho \bar{v} D}$$

$$= 16\frac{1}{\dfrac{\rho \bar{v} D}{\mu_e}}$$

or

$$f = \frac{16}{Re} \tag{5.48}$$

where $(Re) = \rho \bar{v} D/\mu_e$. In other words, for laminar flow the friction factor is inversely proportional to the Reynolds number only.

It was shown earlier that in turbulent flow the velocity profile is approximately uniform across the pipe cross-section and only layers adjacent to the pipe wall will feel the effects of wall roughness. If the pipe roughness is of small magnitude, such that the pipe protuberances penetrate a small part of the laminar sublayer, then the friction factor is independent of pipe roughness. As the length of the protuberances increases, the whole of the laminar sublayer and adjacent layers begin to feel the effects of pipe wall roughness. The pipe wall roughness, ε, is measured by the roughness height or protuberance length, and is expressed as a dimensionless quantity by dividing ε by the pipe diameter.

In drilling engineering it would be extremely difficult to routinely measure pipe wall roughness while the pipe is in service, owing to the hostile environment in which the equipment is used and to the practical difficulties in carrying out this measurement. Because of these difficulties and the need to arrive at simple and practical equations for determining turbulent pressure losses, the following relationships can be used to determine the friction factor, f.

Blasius equation Schlumberger (1984):

$$f = 0.057(Re)^{-0.2} \tag{5.49}$$

Prandtl equation:

$$1/\sqrt{f} = 4\log[(Re)\sqrt{f}] + 0.8 \tag{5.50}$$

Equation (5.49) is accurate up to $(Re) = 10^5$, which is more than sufficient for practical drilling engineering problems. Equation (5.50) is somewhat difficult to solve and normally only provides marginally accurate results for (Re) larger than 10^5.

For a fuller treatment of turbulent flow the reader is advised to consult the textbook by Skelland[1].

The turbulent flow equation will first be derived in consistent units and then converted to field units for the appropriate system.

Recall

$$(Re) = \frac{\rho V D}{\mu} \tag{5.5}$$

Combining Equations (5.5) and (5.49) yields

$$f = 0.057\left(\frac{\rho v D}{\mu}\right)^{-0.2} \tag{5.51}$$

Substituting Equation (5.51) in Equation (5.46) yields

$$\Delta p = 2(0.057)\left(\frac{\rho v D}{\mu}\right)^{-0.2}\frac{L\rho v^2}{D}$$

Simplifying further,

$$\Delta p = (2 \times 0.057)\frac{L\rho^{0.8} v^{1.8}\mu^{0.2}}{D^{1.2}} \tag{5.52}$$

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The following substitution will be made in the above equation:

$$Q = vA, \; v = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}$$

or

$$v = \frac{4Q}{\pi D^2} \qquad (5.53)$$

Also, turbulent viscosity, μ, is normally taken to be equal to PV/3.2. Making substitutions for v and μ in Equation (5.52) yields

$$p = (2 \times 0.057)\left(\frac{4}{\pi}\right)^{1.8}(3.2)^{-0.2} \times \frac{L\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2}}{D^{4.8}} \qquad (5.54)$$

Equation (5.54) is in consistent units and must be converted to field units.

Field units for Equation (5.54)

Metric units

$$p = (2 \times 0.057)\left(\frac{4}{\pi}\right)^{1.8}(3.2)^{-0.2}L\,(\mathrm{m})$$

$$\times \rho^{0.8}\left[\frac{\mathrm{kg}}{\mathrm{l}} \times \left(\frac{1000\ \mathrm{l}}{\mathrm{m}^3}\right)\right]^{0.8}$$

$$\times Q^{1.8}\left(\frac{\mathrm{l}}{\mathrm{min}} \times \frac{\mathrm{m}^3}{1000\ \mathrm{l}} \times \frac{\mathrm{min}}{60\ \mathrm{s}}\right)^{1.8}$$

$$\times pv^{0.2}\left(\mathrm{cP} \times \frac{10^{-3}\frac{\mathrm{kg}}{\mathrm{ms}}}{\mathrm{cP}}\right)^{0.2}$$

$$\div D^{4.8}\left(\mathrm{mm} \times \frac{\mathrm{m}}{1000\ \mathrm{m}}\right)^{4.8}$$

Hence,

$$\text{constant} = 2 \times (0.057)\left(\frac{4}{\pi}\right)^{1.8}(3.2)^{-0.2}$$

$$\times (1000)^{(0.8-1.8+4.8)} \times (60)^{-1.8}$$

$$\times (10^{-3})^{0.2}$$

$$= 5\,546\,778$$

$$\text{dimensions} = (\mathrm{m})^1(\mathrm{kg})^{0.8}(\mathrm{m})^{-2.4}(\mathrm{m})^{5.4}(\mathrm{s})^{-1.8}(\mathrm{kg})^{0.2} \times (\mathrm{m})^{-0.2}(\mathrm{s})^{-0.2}(\mathrm{m})^{-4.8}$$

$$= \frac{\mathrm{kg}}{\mathrm{m\,s}} = \frac{1}{\mathrm{m}^2}\left(\frac{\mathrm{kg\,m}}{\mathrm{s}}\right) = \mathrm{N/m^2}\ (\text{or Pa})$$

Hence,

$$p = 5\,546\,778\frac{\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2}L}{D^{4.8}}\ \mathrm{N/m^2}$$

$$\Delta p = 55.5\frac{\rho^{0.8}Q^{1.8}(\rho v)^{0.2}L}{D^{4.8}}\ \mathrm{bar} \qquad (5.55)$$

Imperial units In Imperial units Equation (5.54) becomes

$$p = \frac{8.9 \times 10^{-5}\,\rho^{0.8}\,Q^{1.8}\,(\mathrm{PV})^{0.2}\,L}{D^{4.8}} \qquad (5.56)$$

Units of Equation (5.56) are: ρ = ppg, Q = gpm, PV = cP, L = ft, D = in.

For annular flow, Equations (5.55) and (5.56) should be modified as follows.

From Equations (5.46) and (5.51),

$$\Delta p = 0.114\left(\frac{\rho v(D_h - D_p)}{\mu}\right)^{-0.2}\frac{L\rho V^2}{D_h - D_p}$$

$$= 0.114\frac{\rho^{0.8}V^{1.8}(\mu)^{0.2}L}{D_h - D_p^{1.2}} \qquad (5.57)$$

Substituting the following equations in Equation (5.57):

$$\mu = \frac{PV}{3.2}\ (\mu = \text{turbulent fluid viscosity})$$

and

$$V = \frac{Q}{A} = \frac{4}{\pi}\frac{Q}{(D_h^2 - D_p^2)} = \frac{4}{\pi}\frac{Q}{(D_h - D_p)(D_h + D_p)}$$

yields

$$\Delta p = 0.114\left(\frac{4}{\pi}\right)^{1.8}(3.2)^{-0.2}\rho^{0.8}(PV)^{0.2}Q^{1.8}$$

$$\times \left[\frac{1}{(D_h - D_p)(D_h + D_p)}\right]^{1.8} \times \frac{1}{(D_h - D_p)^{1.2}} \times L$$

Therefore,

$$\Delta p = 0.114\left(\frac{4}{\pi}\right)^{1.8}(3.2)^{-0.2} \times \frac{\rho^{0.8}(PV)^{0.2}Q^{1.8}L}{(D_h - D_p)^3(D_h + D_p)^{1.8}} \qquad (5.58)$$

Using metric field units and simplifying gives

$$\Delta p = 55.5\frac{\rho^{0.8}Q^{1.8}(\rho v)^{0.2}L}{(D_h - D_p)^3(D_h + D_p)^{1.8}} \qquad (5.59)$$

(Δp is in bars.)

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In Imperial field units

$$\Delta p = \frac{8.91 \times 10^{-5} \rho^{0.8} Q^{1.8} (\rho v)^{0.2}}{(D_h - D_p)^3 (D_h + D_p)^{1.8}} \text{ psi} \qquad (5.60)$$

FLOW THROUGH NOZZLES

Tri-cone bits are normally equipped with two or three nozzles to direct the drilling fluid at high velocity to the bottom of hole in order to lift the cuttings up the hole and also to clean the teeth of the bit. Owing to the small area of a nozzle, fluid velocity through the nozzle is normally high and the flow pattern is always turbulent.

Figure 5.11 gives a cross-section of a typical nozzle. Thus, a nozzle can be approximated as an orifice enabling Bernoulli's equation to be applied at points 1 and 2 of Figure 5.11:

$$\left(p_1 + \frac{1}{2}\rho v_1^2\right)_{\text{point 1}} = \left(p_2 + \frac{1}{2}\rho v_2^2\right)_{\text{point 2}}$$

$$(p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

When $A_2 < A_1$, $v_2^2 \gg v_1^2$ and the above equation simplifies to

$$\Delta p = \frac{1}{2}\rho v_2^2$$

where $\Delta p = p_1 - p_2$

$$v_2 = \sqrt{\frac{2\Delta p}{\rho}} \qquad (5.61)$$

Equation (5.61) does not consider frictional loss in the nozzle and a correction factor known as the coefficient of discharge, C_d, is normally introduced to take account of this frictional loss. Hence,

$$v_2 = C_d \sqrt{\frac{2\Delta p}{\rho}} \qquad (5.62)$$

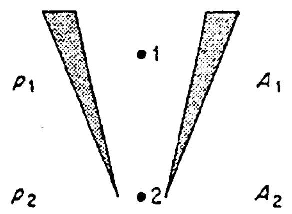

Fig. 5.11. Fluid flow through a nozzle.

The value of $C_d \simeq 0.95$, and Equation (5.62) becomes

$$V_n = 0.95\sqrt{\frac{2\Delta p}{\rho}} = 1.344\sqrt{\frac{\Delta p}{\rho}} \qquad (5.63)$$

where V_n = nozzle velocity or jet velocity.

Rearranging Equation (5.63) to express Δp in terms of V_n and ρ, Equation (5.64) is obtained:

$$\Delta p = \frac{V_n^2 \times \rho}{1.805} \qquad (5.64)$$

Equation (5.63) in Imperial units

$$V_n(\text{ft/s}) = 1.344 \sqrt{\frac{\Delta p\left(\frac{\text{lb}}{\text{in}^2}\,\frac{144\text{ in}^2}{\text{ft}^2}\right)}{\rho\left(\frac{\text{lbm}}{\text{gal}} \times \frac{7.48\text{ gal}}{\text{ft}^3}\right)}}$$

$$V_n = 5.897\sqrt{\text{ft}^3 \times \frac{1}{\text{ft}^3} \times 32\frac{\text{ft}^2}{\text{s}^2}\left(\frac{\Delta p}{\rho}\right)}$$

$$V_n = 33.3585\sqrt{\frac{\Delta p}{\rho}} \text{ ft/s} \qquad (5.65)$$

where Δp is in lbf/in^2 and ρ is in lbm/gal), or

$$\Delta p = \frac{\rho V_n^2}{1113} \text{ psi} \qquad (5.66)$$

Equation (5.63) in metric units

$$V_n = 1.344\sqrt{\frac{\text{bar}\left(\frac{10^5\text{ N/m}^2}{\text{bar}}\right)}{\frac{\text{kg}}{\text{l}}\left(\frac{1000\text{ l}}{\text{m}^3}\right)}} \text{ m/s} \qquad (5.67)$$

$$V_n = 13.44\sqrt{\frac{\Delta p}{\rho}}$$

or

$$\Delta p = \frac{\rho V_n^2}{180.6} \text{ bar} \qquad (5.68)$$

Nozzle area

A tri-cone bit is normally equipped with three nozzles and the objective of the hydraulic programme is to determine the total area and size of nozzles. Using $Q = VA$,

$$A_T = \frac{Q}{V_n} \qquad (5.69)$$

where A_T = total area of nozzles.

(P. 78)

Field units of Equation (5.69)

Imperial units

$$A_T(\text{in}^2) = \frac{Q\dfrac{\text{gal}}{\text{min}} \times \left(\dfrac{\text{ft}^3}{7.48\ \text{gal}}\right)\left(\dfrac{\text{min}}{60\ \text{s}}\right)}{V_n(\text{ft/s})}\left(\frac{144\ \text{in}^2}{\text{ft}^2}\right)$$

$$A_T = \frac{0.32Q}{V_n}\ \text{in}^2 \tag{5.70}$$

where Q is in gal/min and V_n in ft/s. For three nozzles

$$A_T = 3A = 3\frac{\pi}{4}d^2$$

or

$$d = \sqrt{\frac{4A_T}{3\pi}} \tag{5.71}$$

where d is size of nozzle in inches.

Normally, a nozzle size is expressed in multiples of $\frac{1}{32}$ of an inch; an open nozzle is given size 32 or 1 in (24.5 mm). Thus, Equation (5.71) can be modified to give nozzle sizes in multiples of $\frac{1}{32}$ as follows:

$$d_N = 32\sqrt{\frac{4A_T}{3\pi}} \tag{5.72}$$

Metric units

$$A_T(\text{mm}^2) = \frac{\left(Q\dfrac{\text{l}}{\text{min}} \times \dfrac{\text{m}^3}{1000\ \text{l}} \times \dfrac{\text{min}}{60\ \text{s}}\right)}{V_n(\text{m/s})} \times \left(\frac{10^6\ \text{mm}^2}{\text{m}^2}\right)$$

$$A_T = \frac{1000Q}{60V_n}\ \text{mm}^2$$

where A_T is area of three nozzles. Also,

$$d = \sqrt{\frac{4A_T}{3\pi}}\ \text{mm} \tag{5.73}$$

As manufacturers specify nozzle sizes in multiples of $\frac{1}{32}$ of an inch, Equation (5.73) should first be converted to inches and then multiplied by 32 to obtain multiples of 32. Thus,

$$d = \frac{1}{25.4}\sqrt{\frac{4A_T}{3\pi}}\ \text{in}$$

where 1 in = 25.4 mm, nozzle size, $d_N = d \times 32$ and

$$d_N = 1.2598\sqrt{\frac{4A_T}{3\pi}} \tag{5.74}$$

Units of A in Equation (5.74) are still mm² but the size of nozzle is expressed in multiples of $\frac{1}{32}$ in.

Example 5.3

Determine the nozzle sizes in multiples of 32 to be used in a three-cone bit when 500 gpm of mud is circulated at a pressure drop of 1000 psi through the bit. The mud density is 10 ppg.

Solution

From Equation (5.66),

$$\Delta p = \frac{\rho V_n^2}{1113}$$

or

$$V_n = 33.3585\sqrt{\frac{\Delta p}{\rho}}$$

$$= 33.3585\sqrt{\frac{1000}{10}}$$

$$V_n = 333.58\ \text{ft/s}\ (101.7\ \text{m/s})$$

From Equation (5.70),

$$A_T = \frac{0.32Q}{V_n} = \frac{0.32 \times 500}{333.58}$$

$$= 0.4796\ \text{in}^2\ (309.4\ \text{mm}^2)$$

From Equation (5.72),

$$d_N = 32\sqrt{\frac{4A_T}{3\pi}}$$

$$d_N = 14.44\ \text{in}\ (353.8\ \text{mm})$$

Nozzle sizes come only in integer values of $\frac{1}{32}$ in. The above result indicates that two nozzles of size 14 and one of size 15 should be used. This can be checked as follows:

$$A_T = 2 \times \text{area of } \frac{14}{32} \text{ nozzle} + \text{area of } \frac{15}{32} \text{ nozzle}$$

$$= 2\left[\frac{\pi}{4}\left(\frac{14}{32}\right)^2\right] + \frac{\pi}{4}\left(\frac{15}{32}\right)^2$$

$$A_T = 0.3007 + 0.1726$$

$$= 0.4733\ \text{in}^2\ (\text{approximately equal to } 0.4796)$$

If two 15/32 nozzles were used instead of one, then

(p. 79)

the total area A_T becomes:

$$A_T = 2 \times \frac{\pi}{4}\left(\frac{15}{32}\right)^2 + \frac{\pi}{4}\left(\frac{14}{32}\right)^2$$

$$= 0.4954 \text{ in}^2$$

This is greater than 0.4796 in^2 calculated from Equation (5.72). Therefore, two size 14 and one size 15 should be used.

References

1. Skelland, A. H., 1967. *Non-Newtonian Flow and Heat Transfer*. John Wiley, New York.
2. Schlumberger, 1984. *Cementing Technology*. Dowell Schlumberger Publications

Problem

Prove that the effective viscosity in field units for an annular flow obeying a power-law model is given by

$$\mu_e = 200K(D_h - D_p)\left[\frac{0.8}{(D_h - D_p)}\left(\frac{2n+1}{n}\right)\right]^n V^{n-1} \text{ (Imperial units)}$$

and

$$\mu_e = 0.04K(D_h - D_p)\left[\frac{4000}{(D_h - D_p)}\left(\frac{2n+1}{n}\right)\right]^n V^{n-1} \text{ (metric units)}$$

(p. 80)

Chapter 7

Rig Hydraulics

INTRODUCTION

Proper utilisation of hydraulic energy at the drill bit and the determination of pressure losses at the various parts of the drill string are some of the topics which could be discussed under the heading, 'Rig Hydraulics'. Several models exist for the calculation of pressure losses in pipes and annuli. Each model is based on a set of assumptions which cannot be completely fulfilled in any drilling situation.

This chapter will cover the following topics:

Pressure losses
Surface connection losses
Pipe and annular losses
Pressure drop across bit
Optimisation of bit hydraulics
Surface pump pressure
Hydraulic criteria
Comparison of BHHP and IF criteria
Nozzle selection
Optimum flow rate
Field method of optimising bit hydraulics
A practical check on the efficiency of the bit hydraulics programme

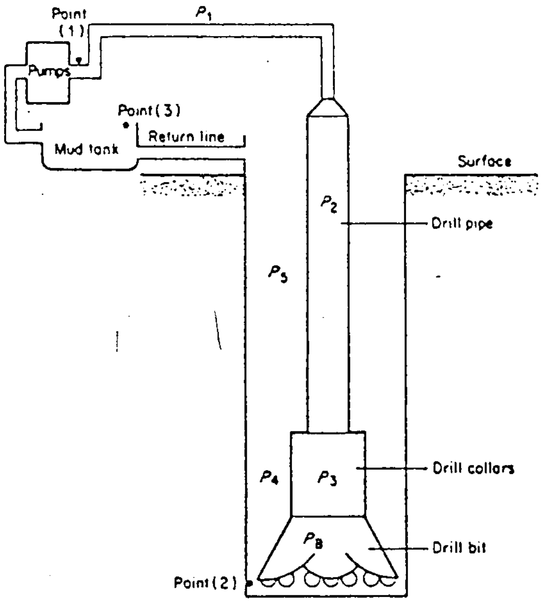

Fig. 7.1. Schematic drawing of the circulating system. Points (1) and (3) are assumed to be at the same level.

PRESSURE LOSSES

Consider the schematic drawing in Figure 7.1, where the various parts of the drill string, drill bit and surface connections are included. Owing to frictional forces, the circulating system will lose energy when fluid is pumped from point (1) to point (2) and back to point (3) in the mud tank. Therefore, the first objective of rig hydraulics is to calculate pressure losses resulting from frictional forces in each part of the circulating system.

Such pressure losses in the various parts of the circulating system can conveniently be discussed under four headings: (1) surface connection losses; (2) pipe losses; (3) annular losses; and (4) losses across bit. These pressure losses depend upon the type of fluid used and the type of flow in the circulating system.

(P. 81)

SURFACE CONNECTION LOSSES (P_1)

Pressure losses in surface connections (P_1) are those taking place in standpipe, rotary hose, swivel and kelly. The task of estimating surface pressure losses is complicated by the fact that such losses are dependent on the dimensions and geometries of surface connections. These dimensions can vary with time, owing to continuous wear of surfaces by the drilling fluids. The following general equation may be used to evaluate pressure losses in surface connections:

$$P_1 = E\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2} \text{ psi} \quad (7.1)$$

or

$$P_1 = E\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2} \text{ bar} \quad (7.1a)$$

where ρ = mud weight (lbm/gal, or kg/l); Q = volume flow rate (gpm, or l/min); E = a constant depending on type of surface equipment used; and PV = plastic viscosity (cP).

In practice, there are only four types of surface equipment; each type is characterised by the dimensions of standpipe, kelly, rotary hose and swivel. Table 7.1 summarises the four types of surface equipment.

The values of the constant E in Equations (7.1) and (7.1a) are given in Table 7.2.

PIPE AND ANNULAR LOSSES

Pipe losses take place inside the drillpipe and drill collars, and are designated in Figure 7.1 as P_2 and P_3, respectively. Annular losses take place around the drill collars and drillpipe, and are designated as P_4 and P_5 in Figure 7.1. The magnitudes of P_2, P_3, P_4 and P_5 depend on: (a) dimensions of drillpipe (or drill collars), e.g. inside and outside diameter and length; (b) mud rheological properties, which include mud weight, plastic viscosity and yield point; and (c) type of flow, which may be laminar, plug or turbulent.

It should be noted that the actual behaviour of drilling fluids downhole is not accurately known and

TABLE 7.2 Values of the constant E

Surface equipment type	Value of E	
	Imperial units	Metric units
1	2.5×10^{-4}	8.8×10^{-6}
2	9.6×10^{-5}	3.3×10^{-6}
3	5.3×10^{-5}	1.8×10^{-6}
4	4.2×10^{-5}	1.4×10^{-6}

fluid properties measured at the surface usually assume different values at bottom hole conditions. Several models exist for the calculation of pressure losses, each producing different values for the same existing conditions.

Only two models will be used here: the Bingham plastic model and the power-law model. In Chapter 5 all the necessary equations were derived and Tables 7.3 and 7.4 summarise these equations.

PRESSURE DROP ACROSS BIT

The objective of any hydraulics programme is to optimise pressure drop across the bit such that maximum cleaning of bottom hole is achieved.

For a given length of drill string (drillpipe and drill collars) and given mud properties, pressure losses P_1, P_2, P_3, P_4 and P_5 will remain constant. However, the pressure loss across the bit is greatly influenced by the sizes of nozzles used, and the latter determine the amount of hydraulic horsepower available at the bit. The smaller the nozzle the greater the pressure drop and the greater the nozzle velocity. In some situations where the rock is soft to medium in hardness, the main objective is to provide maximum cleaning and not maximum jetting action. In this case a high flow rate is required with bigger nozzles. These points are discussed later under 'Optimisation of Bit Hydraulics' (page 141).

TABLE 7.1 Four types of surface equipment

Surface equipment type	Standpipe		Rotary hose		Swivel		Kelly	
	Length (ft)	ID (in)	Length (ft)	ID (in)	Length (ft)	ID (in)	Length (ft)	ID (in)
1	40	3.0	40	2.0	4	2.0	40	2.25
2	40	3.5	55	2.5	5	2.5	40	3.25
3	45	4.0	55	3.0	5	2.5	40	3.25
4	45	4.0	55	3.0	6	3.0	40	4.00

(P. 89)

TABLE 7.3 Summary of equations (Imperial units)

Bingham plastic model

Pipe flow

(1) Determine average velocity:

$$\bar{V} = \frac{24.5Q}{D^2}\ \text{ft/min}$$

(2)
$$V_c = \frac{97\,\text{PV} + 97\sqrt{(\text{PV})^2 + 8.2\rho D^2\,\text{YP}}}{\rho D}\ \text{ft/min}$$

(3) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{8.91 \times 10^{-5}\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{D^{4.8}}\ \text{psi}$$

If $\bar{V} < V_c$, flow is laminar; use

$$P = \frac{L}{300D}\left[\text{YP} + \frac{(\text{PV})\bar{V}}{5D}\right]\text{psi}$$

Annular flow

(1) Determine average velocity:

$$\bar{V} = \frac{24.5Q}{D_h^2 - OD_p^2}\ \text{ft/min}$$

(2) Determine V_c from

$$V_c = \frac{97\,\text{PV} + 97\sqrt{(\text{PV})^2 + 6.2\rho D_e^2\,\text{YP}}}{\rho D_e}\ \text{ft/min}$$

where $D_e = D_h - OD_{dp}$ (or OD_{dc})

(3a) If $\bar{V} > V_c$ flow is turbulent; use

$$P = \frac{8.91 \times 10^{-5}\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{(D_h - OD)^3(D_h + OD)^{1.8}}\ \text{psi}$$

where OD is the outside diameter of drill pipe or drill collars.

(3b) If $\bar{V} < V_c$, flow is laminar; use

$$P = \frac{L\,\text{PV}\,\bar{V}}{60\,000D_e^2} + \frac{L\,\text{YP}}{200D_e}\ \text{psi}$$

where D_e is the annular distance and $\bar{V}$ is the average velocity (ft/min).

Pressure loss across bit

(1) From previous calculations, determine pressure drop across the bit, using

$$P_{bit} = P_{standpipe} - (P_{dp} + P_{dc} + P_{adp} + P_{adc})$$

(2) Determine nozzle velocity (ft/s) from

$$V_n = 33.36\sqrt{\frac{P_{bit}}{\rho}}$$

(3) Determine total area of nozzles from

$$A = 0.32\frac{Q}{V_n}\ \text{in}^2$$

(4) Determine nozzle sizes in multiples of 32 from

$$d_N = \left(\sqrt{\frac{4A}{3\pi}}\right) \times 32$$

Power-law model

Pipe flow

(1) Determine n and K:

$$\theta_{600} = 2\,\text{PV} + \text{YP} \quad \theta_{300} = \text{PV} + Y$$

$$\boxed{\begin{aligned}\text{PV} &= \theta_{600} - \theta_{300}\\ \text{YP} &= 2\theta_{300} - \theta_{600}\end{aligned}}$$

$$n = 3.32\log\left(\frac{\theta_{600}}{\theta_{300}}\right),\quad K = \frac{\theta_{300}}{(511)^n}$$

(2)
$$V_c = \left[\frac{5.82(10^4)K}{\rho}\right]^{1/(2-n)} \cdot \left[\frac{1.6}{D}\,\frac{3n+1}{4n}\right]^{n/(2-n)}\ \text{ft/min}$$

$$\bar{V} = \frac{24.5Q}{D^2}\ \text{ft/min}$$

(3) If $\bar{V} > V_c$ flow is turbulent; use:

$$P = \frac{8.91(10^{-5})\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{D^{4.8}}\ \text{psi}$$

If $\bar{V} < V_c$, flow is laminar; use:

$$P = \left[\frac{1.6\bar{V}}{D}\,\frac{(3n+1)}{4n}\right]^n \frac{KL}{300D}\ \text{psi}$$

Annular flow

(1) Determine n and K as above

(2)
$$V_c = \left[\frac{3.878(10^4)K}{\rho}\right]^{1/(2-n)} \times \left[\frac{2.4}{D_h - OD_p}\left(\frac{2n+1}{3n}\right)\right]^{n/(2-n)}\ \text{ft/min}$$

$$\bar{V} = \frac{24.5Q}{D_h^2 - OD_p^2}$$

(3) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{8.91(10^{-5})\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{(D_h - OD)^3(D_h + OD)^{1.8}}\ \text{psi}$$

If $\bar{V} < V_c$, flow is laminar; use

$$P = \left[\frac{2.4\bar{V}}{(D_h - OD)}\,\frac{(2n+1)}{3n}\right]^n \frac{KL}{300(D_h - OD)}\ \text{psi}$$

Pressure loss across bit

Equations are similar to those of the Bingham plastic model.

Field units in Imperial units:
OD = outside diameter (in)
D = inside diameter (in)
L = length (ft)
ρ = density lbm/gal (ppg)
V = velocity (ft/s)
PV = viscosity (cP)
YP = yield point (lbf/100 ft²)

(P-83)

TABLE 7.4 Summary of hydraulics equations: metric units

Pipe flow

Bingham plastic model

(1) Determine average velocity, $\bar{V}$:

$$\bar{V} = 21.2(Q/D^2)\ \text{m/s}$$

Q in l/min; D in mm.

(2) Determine critical velocity, V_c:

$$\checkmark\ V_c = 1.5\left(\frac{\text{PV} + \sqrt{(\text{PV})^2 + 0.1064\rho D^2\,\text{YP}}}{\rho D}\right)\ \text{m/s}$$

(3a) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{55.5\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{D^{4.8}}\ \text{bar}$$

(3b) If $V < V_c$, flow is laminar; use

$$\checkmark\ P = \frac{32 \times 10^{-2} L\,\text{PV}\,\bar{V}}{D^2} + \frac{2.55\,\text{YP} \times 10^{-2}}{D/L}\ \text{bars}$$

Power-law model

(1) Determine n and K from

$$n = 3.32 \log \frac{\theta_{600}}{\theta_{300}},\quad K = \frac{\theta_{300}}{(511)^n}$$

(2) Determine average velocity, V, and critical velocity, V_c, from

$$\bar{V} = 21.2\left(\frac{Q}{d^2}\right)\ \text{m/s}$$

and

$$V_c = \left[\frac{0.179K}{\rho}\right]^{1/(2-n)}\left[\frac{2000}{D}\left(\frac{3n+1}{n}\right)\right]^{n/(1-n)}\ \text{m/s}$$

(3a) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{55.5\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{D^{4.8}}\ \text{bar}$$

(3b) If $\bar{V} < V_c$, flow is laminar; use

$$P = \frac{19.16 \times 10^{-3} KL}{D}\left[\frac{2000\bar{V}}{D}\left(\frac{3n+1}{4n}\right)\right]^n\ \text{bars}$$

Annular flow

Bingham plastic model

(1) Determine average velocity

$$\bar{V} = \frac{24.5}{D_h^2 - OD_p^2}\ \text{m/s}$$

(2) Determine critical velocity, V_c:

$$V_c = 1.5\left(\frac{\text{PV} + \sqrt{(\text{PV})^2 + 0.079\rho D_e^2\,\text{YP}}}{\rho D_e}\right)$$

where $D_e = D_h = OD_p$

(3a) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{55.5\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{(D_h - OD_p)^3(D_h + OD_p)^{1.8}}\ \text{bars}$$

(3b) If $V_c < \bar{V}$, flow is laminar; use

$$P = \frac{48 \times 10^{-2} L\,\text{PV}\,\bar{V}}{(D_h - OD_p)^2} + \frac{2.87 \times 10^{-2} L\,\text{YP}}{(D_h - OD_p)}\ \text{bars}$$

Power-law model

(1) Determine n and K as above.

(2) Determine average velocity, $\bar{V}$, and critical velocity, V_c, from

$$\bar{V} = 21.2\left(\frac{Q}{D_h^2 - OD_p^2}\right)\ \text{m/s}$$

$$V_c = \left[\frac{0.119K}{\rho}\right]^{1/(2-n)}\left[\frac{4000}{(D_h - OD_p)}\left(\frac{2n+1}{n}\right)\right]^{n/(2-n)}\ \text{m/s}$$

(3a) If $\bar{V} > V_c$, flow is turbulent; use

$$P = \frac{55.5\rho^{0.8}Q^{1.8}(\text{PV})^{0.2}L}{(D_h - OD_p)^3(D_h + OD_p)^{1.8}}\ \text{bars}$$

(3b) If $\bar{V} < V_c$, flow is laminar; use

$$P = \frac{18.9 \times 10^{-3} KL}{(D_h - OD_p)}\left[\frac{4000\,\bar{V}}{(D_h - OD_p)}\frac{(2n+1)}{n}\right]^n\ \text{bars}$$

Pressure loss across bit

Bingham plastic model

(1) From previous calculations, determine the available pressure drop across the bit as follows:

$$P_{bit} = P_{standpipe} - (P_{dp} + P_{dc} + P_{adp} + P_{adc})$$

(2) Determine nozzle velocity from

$$V_n = 13.44\sqrt{\frac{P_{bit}}{\rho}}\ \text{m/s}$$

(3) Determine total area of nozzles from

$$A = \frac{100}{6}\frac{Q}{V_n}\ \text{mm}^2$$

(4) Determine nozzle sizes in multiples of 32 from

$$d_N = 1.2598\sqrt{\frac{4A}{3\pi}}$$

(*Note*: $\mu = (\text{PV})/3.2$ for fully developed turbulent flow.)

Power-law model

Equations are similar to those of the Bingham plastic model.

Field units:
OD = outside diameter
D = inside diameter (mm)
L = length (m)
ρ = density (kg/l)
V = velocity (m/s)
PV = viscosity (cP)
YP = yield point (lbf/100 ft)^2 (as obtained from a viscometer)

(P-84)

To determine the pressure drop across the bit, add the total pressure drops across the system, i.e. $P_1 + P_2 + P_3 + P_4 + P_5$, to give a total value of P_c (described as the system pressure loss). Then determine the pressure rating of the pump used. If this pump is to be operated at, say, 80–90% of its rated value, then the pressure drop across the bit is simply pump pressure minus P_c.

The equations necessary for the calculation of jet velocity, pressure drop across bit and nozzle sizes are given in Tables 7.3 and 7.4.

Example 7.1

Using the Bingham plastic and power-law models, determine the various pressure drops, nozzle velocity and nozzle sizes for a section of 12.25 in (311 mm) hole. Two pumps are used to provide 700 gpm (2650 l/min).

Data:

plastic velocity	= 12 cP
yield point	= 12 lb/100 ft^2
	= (0.479 × 12) N/m^2
mud weight	= 8.824 lb/gal (1.057 kg/l)
drillpipe ID	= 4.276 in (108.6 mm)
OD	= 5 in (127 mm)
length	= 6480 ft (1975 m)
drill collars ID	= 2.875 in (73 mm)
OD	= 8 in (203 mm)
length	= 620 ft (189 m)

Last casing was $13\frac{3}{8}$ in (340 mm) with an ID of 12.565 in (319 mm). $13\frac{3}{8}$ in casing was set at 2550 ft (777 m). The two pumps are to be operated at a maximum standpipe pressure of 2200 psi (151.7 bar). Assume a surface equipment type of 4.

Solution

The solution to this example will be presented in Imperial units, with a summary of the Bingham plastic solution in metric units.

1. *Bingham plastic model*

Surface losses From Equation (7.1), pressure losses in surface equipment P_1 are given by

$$P_1 = E\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2}$$

From Table 7.2, the value of the constant E for type 4 is 4.2×10^{-5}; hence, Equation (7.1) becomes

$$P_1 = 4.2 \times 10^{-5}\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2}$$
$$= 4.2 \times 10^{-5}(8.824)^{0.8}(700)^{1.8}(12)^{0.2}$$
$$= 52 \text{ psi}$$

Pipe losses

Pressure losses inside drillpipe

$$\text{Average velocity } \bar{V} = \frac{24.5Q}{D^2} = \frac{24.5 \times 700}{(4.276)^2}$$
$$= 937.97 \text{ ft/min}$$

Critical velocity

$$V_c = \frac{97\,\mathrm{PV} + 97\sqrt{(\mathrm{PV})^2 + 8.2\rho D^2\,\mathrm{YP}}}{\rho D} \text{ ft/min}$$
$$= \frac{97 \times 12 + 97\sqrt{(12)^2 + 8.2 \times 8.824 \times (4.276)^2 \times 12}}{8.824 \times 4.276}$$
$$= 356 \text{ ft/min}$$

Since $\bar{V} > V_c$, flow is turbulent and pressure drop inside drill pipe is calculated from:

$$P_2 = \frac{8.91 \times 10^{-5}\rho^{0.8}Q^{1.8}(\mathrm{PV})^{0.2}L}{D^{4.8}}$$
$$= \frac{8.91 \times 10^{-5}(8.824)^{0.8}700^{1.8}12^{0.2} \times 6480}{(4.276)^{4.8}}$$
$$= 669.9 \text{ psi}$$
$$\approx 670 \text{ psi}$$

Pressure losses inside drill collars Following the same procedure as for drillpipe losses, we obtain

$$\bar{V} = \frac{24.5 \times 700}{(2.875)^2} = 2074.9 \text{ ft/min}$$

$$V_c = \frac{97 \times 12 + 97\sqrt{(12)^2 + 8.2 \times 8.824 \times (2.875)^2 \times 12}}{8.824 \times 2.875}$$

$$V_c = 373 \text{ ft/min}$$

Since $\bar{V} > V_c$, flow is turbulent and pressure loss inside drill collars P_3 is determined from

$$P_3 = \frac{8.91 \times 10^{-5}(8.824)^{0.8}(700)^{1.8}(12)^{0.2} \times 620}{(2.875)^{4.8}}$$
$$= 431 \text{ psi}$$

(P.85)

Annular pressure losses A rough sketch always helps in simplifying the problem. From Figure 7.2 it can be seen that part of the drillpipe is inside the casing and the rest is inside open hole. Hence, pressure loss calculations around the drillpipe must be split into (a) losses around the drillpipe inside the casing and (b) losses around the drillpipe in open hole.

Pressure losses around drillpipe

Cased hole section

$$\bar{V} = \frac{24.5Q}{(\mathrm{ID_c})^2 - (\mathrm{OD_{dp}})^2} = \frac{24.5 \times 700}{(12.565)^2 - (5)^2}$$

$$= 129.1 \text{ ft/min}$$

where the subscripts 'c' and 'dp' refer to casing and drill pipe, respectively.

$$V_c = \frac{97 \times 12 + 97 \times \sqrt{(12)^2 + 6.2 \times (12.565 - 5)^2 \times 8.824 \times 12}}{8.824 \times (12.565 - 5)}$$

$$= 299.6 \text{ ft/min}$$

Since $\bar{V} < V_c$, flow is laminar and the pressure loss around the drillpipe in the cased hole section is determined from:

$$P_a = \frac{L(\mathrm{PV})\bar{V}}{60\,000(\mathrm{ID_c} - \mathrm{OD_{dp}})^2} + \frac{L\,(\mathrm{YP})}{200(\mathrm{ID_c} - \mathrm{OD_{dp}})}$$

where $L = 2550$ ft and $\bar{V}$ is in ft/min.

$$P_a = \frac{2550 \times 12 \times 129.01}{60\,000(12.565 - 5)^2} + \frac{2550 \times 12}{200(12.565 - 5)}$$

$$= 1.15 + 20.22 = 21.4$$

$$P_a \simeq 21 \text{ psi}$$

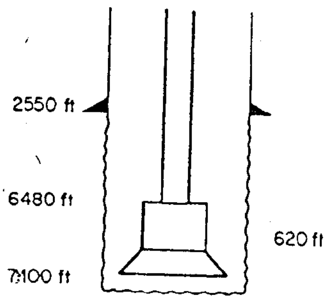

Fig. 7.2. Illustration of annular pressure loss calculation.

Open hole section

$$\bar{V} = \frac{24.5 \times 700}{(12.25)^2 - (5)^2}$$

$$= 137 \text{ ft/min}$$

$$V_c = 300.4 \text{ ft/min}$$

Since $\bar{V} < V_c$, flow is laminar and the pressure loss around the drillpipe in the open-hole section is determined from

$$P_b = \frac{3930 \times 12 \times 137}{60\,000(12.25 - 5)^2} + \frac{3930 \times 12}{200(12.25 - 5)}$$

$$= 35 \text{ psi}$$

(where $L = 6480 - 2550 = 3930$ ft, and L = length of drillpipe in the open-hole section).

Hence, total pressure drop around drillpipe is the sum of P_a and P_b. Thus,

$$P_5 = P_a + P_b = 21 + 35 = 56 \text{ psi}$$

Pressure losses around drill collars

$$\bar{V} = \frac{24.5 \times 700}{(12.25)^2 - (8)^2} = 199.3 \text{ ft/min}$$

$$V_c = \frac{97 \times 12 + 97 \times \sqrt{(12)^2 + 6.2 \times (8.824)(12.25 - 8)^2 \times 12}}{8.824 \times (12.25 - 8)}$$

$$= 314 \text{ ft/min}$$

Since $\bar{V} < V_c$, flow is laminar and pressure loss around drill collars is calculated from:

$$P_4 = \frac{620 \times 12 \times 199.3}{60\,000(12.25 - 8)^2} + \frac{620 \times 12}{200(12.25 - 8)}$$

$$= 1.37 + 8.75 = 10 \text{ psi}$$

Pressure drop across bit Total pressure loss in circulating system, except bit

$$= P_1 + P_2 + P_3 + P_4 + P_5$$

$$= 52 + 670 + 431 + 10 + 56$$

$$= 1219 \text{ psi}$$

Therefore, pressure drop available for bit (P_{bit})

pump pressure $= 2200 - 1219 = 981$ psi

$$\text{Nozzle velocity} = 33.36\sqrt{\frac{P_{\mathrm{bit}}}{\rho}}$$

$$= 33.36\sqrt{\frac{981}{8.824}}$$

$$= 351.7 \text{ ft/s}$$

(P. 86)

Total area of nozzles,

$$A_T = \frac{0.32\,Q}{V_n}$$

$$= \frac{0.32 \times 700}{351.7}$$

$$= 0.6369 \text{ in}^2$$

Nozzle size (in multiples of $\frac{1}{32}$)

$$= 32\sqrt{\frac{4A_T}{3\pi}}$$

$$d_N = 16.64$$

Hence, select two nozzles of size 17 and one of size 16. The total area of these nozzles is 0.6397 in^2, which is slightly larger than the calculated area of 0.6368 in^2. (See also Example 5.3, page 103.)

Summary of Bingham plastic calculations in metric units

$P_1 = 3.49$ bar

$P_2 = 46.22$ bar

$P_3 = 29.77$ bar

$P_4 = 0.68$ bar

$P_5 = 3.74$ bar

$P_{bit} = 151.7 - (P_1 + P_2 + P_3 + P_4 + P_5) = 67.8$ bar

$A_T = 410.32$ mm^2

Nozzle sizes: two 17s and one 16.

2. Power-law model

Surface losses

$$P_1 = 4.2 \times 10^{-5}\rho^{0.8}Q^{1.8}(\text{PV})^{0.2} = 52 \text{ psi}$$

Pipe losses

Pressure losses inside drillpipe (P_2)

$$\theta_{600} = 2(\text{PV}) + \text{YP} = 2(12) + 12 = 36$$

$$\theta_{300} = \text{PV} + \text{YP} = 12 + 12 = 24$$

$$n = 3.32 \log\left(\frac{\theta_{600}}{\theta_{300}}\right) = 0.5846 \simeq 0.585$$

$$K = \frac{\theta_{300}}{(511)^n} = 0.6263 \simeq 0.626$$

$$V_c = \left[\frac{5.82(10^4)K}{\rho}\right]^{[1/(2-n)]}\left[\frac{1.6}{D}\left(\frac{3n+1}{4n}\right)\right]^{[n/(2-n)]}$$

where D = ID of drillpipe and ρ = density of mud.

$$V_c = \left[\frac{5.82 \times 10^4 \times 0.626}{8.824}\right]^{[1/(2-0.585)]} \times \left[\frac{1.6 \times (3 \times 0.585 + 1)}{4.276 \times 4 \times 0.585}\right]^{[0.585/(2-0.585)]}$$

$$= (4128.88)^{0.707}(0.4405)^{0.413}$$

$$= (360.08) \times (0.713)$$

$$= 256.7 \text{ ft/min}$$

$$\bar{V} = \frac{24.5Q}{D^2} = \frac{24.5 \times 700}{(4.276)^2} = 937.97 \text{ ft/min}$$

Since $\bar{V} > V_c$, flow is turbulent and pressure loss inside drill pipe, P_2, is calculated by use of the turbulent flow equation given in Table 7.3:

$$P_2 = \frac{8.91(10^{-5})(8.824)^{0.8}(700)^{1.8}(12)^{0.2}(6480)}{(4.276)^{4.8}}$$

$$= 670 \text{ psi}$$

Pressure losses inside drill collars (P_3) As before, find V_c and $\bar{V}$:

$$V_c = 301 \text{ ft/min}$$

$$\bar{V} = \frac{24.5Q}{D^2} = \frac{24.5 \times 700}{(2.875)^2} = 2074.9 \text{ ft/min}$$

Since $\bar{V} > V_c$, flow is turbulent. Hence, pressure losses inside drill collars, P_3, are

$$P_3 = \frac{8.91(10^{-5})(8.824)^{0.8}(700)^{1.8}(12)^{0.2} \times (620)}{(2.875)^{4.8}}$$

$$P_3 = 431 \text{ psi}$$

Annular losses Because casing has been set, annular losses will have to be determined in the open hole section and cased hole section (see Figure 7.2).

Pressure losses around drillpipe

Cased hole section Values of n and K were calculated above:

$$n = 0.585 \text{ and } K = 0.626$$

$$L = \text{length of cased section} = 2550 \text{ ft}$$

Annular distance = ID$_c$ − OD = inside diameter of 13$\frac{3}{8}$ casing — outside diameter of drillpipe = 12.565 − 5 = 7.565 in

(P. 87)

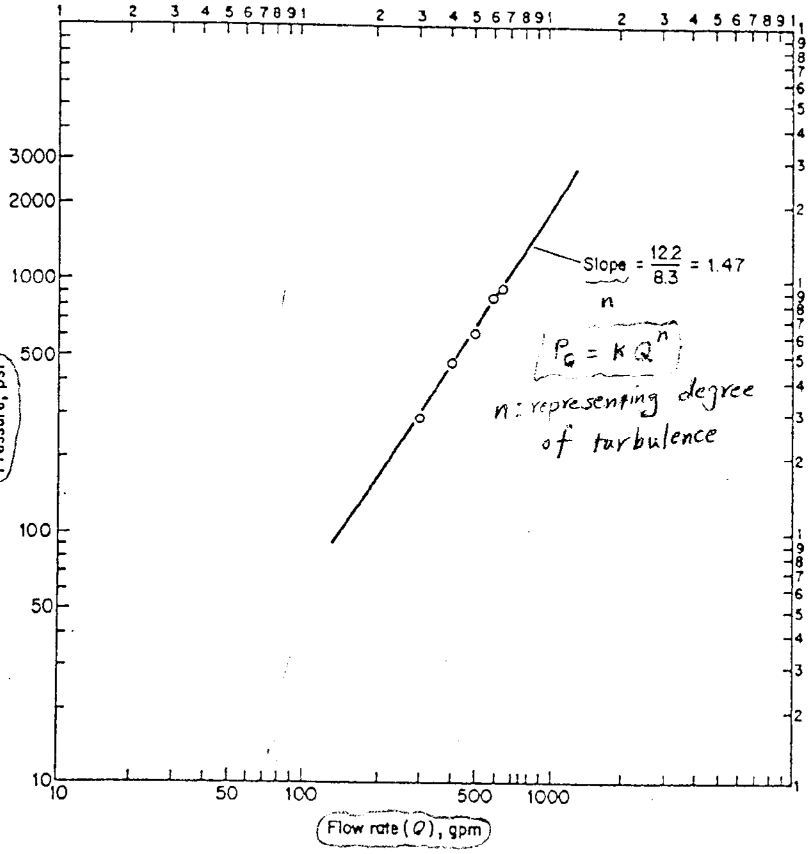

Fig. 7.3. Hydraulics data of Example 2. Maximum surface pressure (P_s) = 2500 psi.

$$V_c = \left[\frac{3.878 \times 10^4 K}{\rho}\right]^{[1/(2-n)]} \times \left[\frac{2.4}{(\mathrm{ID_c} - \mathrm{OD_{dp}})}\left(\frac{2n+1}{3n}\right)\right]^{[n/(2-n)]}$$

$$= 193 \text{ ft/min} \quad (183 \text{ ft/min})$$

$$\bar{V} = \frac{24.5Q}{D_c^2 - (\mathrm{OD_{dp}})^2} = 129 \text{ fpm}$$

Since $\bar{V} < V_c$, flow is laminar and pressure losses around the drillpipe in the cased hole section are given by:

$$P_a = \left[\frac{2.4\bar{V}}{(\mathrm{ID_c} - \mathrm{OD_{dp}})}\left(\frac{2n+1}{3n}\right)\right]^n \frac{KL}{300(\mathrm{ID_c} - \mathrm{OD_{dp}})}$$

$$= \left[\frac{2.4 \times 129}{(12.565 - 5)} \frac{(1.17 + 1)}{3 \times 0.585}\right]^{0.585} \frac{0.626 \times 2550}{300 \times 7.565}$$

$$= 6.98 \text{ psi}$$

$$\approx 7 \text{ psi}$$

Open hole section Length of drillpipe in open hole section = 6480 − 2550 = 3930 ft

Annular distance = hole diameter − OD of drillpipe

= 12.25 − 5 = 7.25 in

$$\bar{V} = \frac{24.5Q}{D^2 - (\mathrm{OD_{dp}})^2} = \frac{24.5 \times 700}{(12.25)^2 - (5)^2} = 137 \text{ fpm}$$

$$V_c = 196 \text{ fpm} \quad (186 \text{ ft/min})$$

Since $\bar{V} < V_c$, flow is laminar and pressure loss around drillpipe in open hole is

$$P_b = \left[\frac{2.4 \times 137}{(12.25 - 5)} \frac{(1.17 + 1)}{3 \times 0.585}\right]^{0.585} \frac{0.626 \times 3930}{300 \times 7.25}$$

$$= 11.9 \text{ psi} \approx 12 \text{ psi}$$

Therefore, total pressure loss around drillpipe is

(P. 88)

given by

$$P_5 = P_s + P_b$$
$$= 7 + 12 = 19 \text{ psi}$$

Pressure losses around drill collars

$$V_c = 231 \text{ ft/min}$$

$$\bar{V} = \frac{24.5 \times Q}{D^2 - OD_{dc}^2} = \frac{24.5 \times 700}{(12.25)^2 - (8)^2} = 199 \text{ fpm}$$

Since $\bar{V} < V_c$, flow is laminar:

$$P_4 = \left[\frac{2.4 \times 199}{(12.25 - 8)}\frac{(1.17 + 1)}{3 \times 0.585}\right]^{0.585} \frac{0.626 \times 620}{300(12.25 - 8)}$$
$$= 5.46 \text{ psi}$$
$$\approx 5 \text{ psi}$$

Pressure drop across bit Pressure losses inside and around the drillpipe and drill collars remain constant so long as: (a) mud properties remain unchanged; and (b) physical dimensions are unchanged. Therefore, total pressure losses through the system except bit

$$= P_1 + P_2 + P_3 + P_4 + P_5$$
$$= 52 + 670 + 431 + 19 + 5$$
$$= 1177 \text{ psi}$$

Therefore,

$$\text{pressure drop across bit} = \text{pump pressure} - 1178$$
$$= 2200 - 1177$$
$$= 1023 \text{ psi}$$

Using the same procedure as presented for the Bingham plastic solution, we obtain

$$\text{nozzle velocity} = V_n = 359 \text{ ft/s}$$

$$\text{total nozzle area, } A_T = 0.6239 \text{ in}^2$$

nozzle size (in multiples of $\frac{1}{32}$) = 16.47. Hence, select two 16s and one 17.

The total area of the nozzles is 0.6144 in^2, which is slightly less than 0.6239 in^2. If two 17s and one 16 were selected, then the total flow area would be 0.6397 in^2, which is slightly larger than the calculated area of 0.6239 in^2. In practice, one opts for the smaller size so that sufficient pressure drop is expended across the bit to give optimum hydraulics.

Calculations in metric units for the power-law model are left as an exercise for the reader.

✓ 3. *Comparison of the two models*

From the above results, it is obvious that the two models produce different nozzle sizes: the Bingham plastic model produced two 17s and one 16, whereas the power-law model produced two 16s and one 17. In practice, this difference is not considered serious, and if the mud pumps are capable of producing more than 2200 psi, then it is likely that three nozzles of size 16 will be chosen.

The reader should note also that the turbulent flow equations presented here use a turbulent viscosity term equal to (PV)/3.2 and not the plastic viscosity. If the plastic viscosity term is used instead, then pressure losses will be 26% higher than those calculated by our turbulent flow equation. It is the author's experience that the use of the turbulent viscosity term (i.e. (PV)/3.2) provides pressure loss values that are in agreement with field results.

OPTIMISATION OF BIT HYDRAULICS

All hydraulics programmes start by calculating pressure drops in the various parts of the circulating system. Pressure losses in surface connections, inside and around the drillpipe, inside and around drill collars, are calculated, and the total is taken as the pressure loss in the circulating system, excluding the bit. This pressure loss is normally given the symbol P_c.

Several hydraulics slide rules are available from bit manufacturers for calculating P_c. The slide rule is not suitable for calculating annular pressure losses, owing to: (a) the fact that annular pressure losses are normally small and may be beyond the scale of the slide rule; and (b) the fact that annular pressures are frequently laminar in nature and most slide rules use turbulent flow models.

A recent paper published in *World Oil*[1] offers a quick method of calculating system pressure losses. A 10% factor is incorporated into the drill string pressure losses to account for annular pressure losses. For shallow wells this method is quite satisfactory; however, for deep wells the annular pressure losses must be determined by using either the power-law model or the Bingham plastic model equations.

SURFACE PRESSURE

The system pressure losses, P_c, having been determined the question is how much pressure drop can be tolerated at the bit (P_{bit}). The value of P_{bit} is controlled entirely by the maximum allowable surface pump pressure.

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Most rigs have limits on maximum surface pressure, especially when high volume rates — in excess of 500 gpm (1893 l/min) — are used. In this case, two pumps are used to provide this high quantity of flow. On land rigs typical limits on surface pressure are in the range 2500–3000 psi for well depths of around 12 000 ft. For deep wells, heavy-duty pumps are used which can have pressure ratings up to 5000 psi.

Hence, for most drilling operations, there is a limit on surface pump pressure, and the criteria for optimising bit hydraulics must incorporate this limitation.

HYDRAULIC CRITERIA

There exist two criteria for optimising bit hydraulics: (1) maximum bit hydraulic horsepower (BHHP); and (2) maximum impact force (IF). Each criterion yields different values of bit pressure drop and, in turn, different nozzle sizes. The engineer is faced with the task of deciding which criterion he is to choose. Moreover, in most drilling operations the circulation rate has already been fixed, to provide adequate annular velocity. This leaves only one variable to optimise: the pressure drop across the bit, P_{bit}. We shall examine the two criteria in detail and offer a quick method for optimising bit hydraulics.

Maximum bit hydraulic horsepower

The pressure loss across the bit is simply the difference between the standpipe pressure and P_c. However, for optimum hydraulics the bit pressure drop must be a certain fraction of the maximum available surface pressure. For a given volume flow rate optimum hydraulics is obtained when the bit hydraulic horsepower assumes a certain percentage of the available surface horsepower.

Surface hydraulic horsepower (HHP_s) is the sum of hydraulic horsepower at bit (BHHP) and hydraulic horsepower in the circulating system (HHP_c). Mathematically this can be expressed as:

$$HHP_s = BHHP + HHP_c$$

or

$$BHHP = HHP_s - HHP_c \tag{7.2}$$

In the case of limited surface pressure, the maximum pressure drop across the bit, as a function of available surface pressure, gives the maximum hydraulic horsepower at the bit for an optimum value of flow rate. In other words, the first term in equation (7.2) must be maximised for maximum BHHP. Equation (7.2) can be written as follows:

$$BHHP = \frac{P_s Q}{1714} - \frac{P_c Q}{1714} \tag{7.2a}$$

where P_s = maximum available surface pressure—also the standpipe pressure as read on the surface gauge (psi); P_c = pressure loss in the circulating system (psi); and Q = volume flow rate (gpm).

The pressure drop in the circulating system, P_c, can be related to Q as

$$P_c = KQ^n \tag{7.3}$$

where K = a constant; and n = index representing degree of turbulence in the circulating system.

Combining Equations (7.2a) and (7.3) gives

$$BHHP = \frac{P_s Q}{1714} - \frac{KQ^{n+1}}{1714} \tag{7.4}$$

Differentiating Equation (7.4) with respect to Q and setting $dBHHP/dQ = 0$ gives

$$P_s = (n+1)KQ^n$$

or

$$P_s = (n+1)P_c \tag{7.5}$$

Also

$$P_c = P_s - P_{bit} \tag{7.6}$$

Substituting Equation (7.6) in Equation (7.5) and simplifying gives

$$P_{bit} = \frac{n}{n+1} P_s \tag{7.7}$$

In the literature several values of n have been proposed, all of which fall in the range 1.8–1.86. Hence, when $n = 1.86$, Equation (7.7) gives $P_{bit} = 0.65\ P_s$. In other words, for optimum hydraulics, the pressure drop across the bit should be 65% of the total available surface pressure.

The actual value of n can be determined in the field by running the mud pump at several speeds and reading the resulting pressures. A graph of $P_c(=P_s - P_{bit})$ against Q is then drawn. The slope of this graph is taken as the index n.

Maximum impact force

In the case of limited surface pressure Robinson[2] showed that, for maximum impact force, the pressure drop across the bit (P_{bit1}) is given by

$$P_{bit1} = \frac{n}{n+2} P_s \tag{7.8}$$

where n = slope of P_c vs. Q; and P_s = maximum available surface pressure.

(P. 90)

Derivation of Equation (7.8) is left as an exercise for the reader (see Problem 1 at the end of this chapter).

The bit impact force (IF) can be shown to be a function of Q and P_{bit} according to the following equation:

$$IF = \frac{Q\sqrt{\rho P_{bit}}}{58} \quad (7.9)$$

$P_{bit} = P_s - P_c$

$P_c = KQ^n$

$P_{bit} = \frac{n}{n+2} P_s$

where ρ = mud weight (ppg).

Comparison of BHHP and IF criteria

The ratio R, of pressure drop across bit, as given by the BHHP criterion, and that as given by the IF criterion (Equations 7.7 and 7.8) is

$$R = \frac{n}{n+1} P_s \div \frac{n}{n+2} P_s$$

or

$$R = \frac{n+2}{n+1} \quad (7.10)$$

Equation (7.10) shows that the pressure drop across the bit, as determined by the maximum BHHP criterion, is always larger than that given by the maximum IF method. For laminar flow, $n = 1$ and R assumes its maximum value of 1.5. Actual field values of n are larger than 1. Results of Table 7.5, which has been constructed for values of n between 1 and 2, indicate that R decreases parabolically with increasing value of n, but can never assume unity. In other words, P_{bit} is always larger than P_{bit1}.

NOZZLE SELECTION

Smaller nozzle sizes are always obtained when the maximum BHHP method is used, as it gives larger values of P_{bit} than those given by the maximum IF method. The following equations may be used to determine total flow area and nozzle sizes:

$$A_T = 0.0096\, Q \sqrt{\frac{\rho}{P_{bit}}} \text{ in}^2 \quad (7.11)$$

$$d_N = 32\sqrt{\frac{4A_T}{3}} \quad (7.12)$$

where A_T = total flow area (in^2); and d_N = nozzle size in multiples of $\frac{1}{32}$ in.

TABLE 7.5 Ratio $P_{bit}/P_{bit\,1}$ as a function of index n

n	$R = \frac{n+2}{n+1}$
1.0	1.50
1.2	1.45
1.5	1.40
1.8	1.36
2.0	1.33

OPTIMUM FLOW RATE

Optimum flow rate is obtained using the optimum value of P_c, n and maximum surface pressure, P_s. For example, using the maximum BHHP criterion, P_c is determined from

$$P_c = P_s - P_{bit} = P_s - \frac{n}{n+1} P_s$$

$$P_c = \frac{1}{n+1} P_s \Rightarrow Q_{opt} \quad (7.13)$$

The value of n is equal to the slope of the P_c–Q graph. The optimum value of flow rate, Q_{opt}, is obtained from the intersection of the P_c value (as determined by Equation 7.13) and the P_c–Q graph (see example 7.2).

FIELD METHOD OF OPTIMISING BIT HYDRAULICS

The index n can only be determined on site and is largely controlled by downhole conditions. The following method for determining n was suggested by Robinson[2] and is summarised here briefly.

√(1) Prior to POH current bit for next bit change, run the pump at four or five different speeds and record the resulting standpipe pressures.

√(2) From current nozzle sizes and mud weight determine pressure losses across the bit for each value of flow rate, using Equation (7.11) or a hydraulics slide rule.

√(3) Subtract P_{bit} from standpipe pressure to obtain P_c.

√(4) Plot a graph of P_c against Q on log–log graph paper and determine the slope of this graph, which is the index n in Equations (7.7), (7.8) and (7.13).

√(5) For the next bit run, Equation (7.7) or (7.8) is used to determine P_{bit} that will produce maximum bit hydraulic horsepower. Nozzle sizes are then selected by use of this value of P_{bit}.

For a particular rig and field the index n will not vary widely if the same drilling parameters are used.

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For standardisation purposes it is recommended that the above test be run at three depths for each bit run. The average value of n for each bit run can then be used for designing optimum hydraulics.

Example 7.2

Prior to changing of bit in a $12\frac{1}{4}$ in hole, the standpipe pressures listed in Table 7.6 were recorded at different flow rates with the present bit still on bottom. Present hole depth is 6528 ft and the next bit is expected to drill down to 8000 ft. Other relevant data are:

present nozzle sizes	= three $\frac{16}{32}$
mud weight	= 8.7 ppg (65 pcf)
current flow rate	= 600 gpm
maximum allowable surface pressure	= 2500 psi

Determine the optimum hydraulic parameters for the next bit run, using BHHP and IF criteria.

Solution

Maximum bit hydraulic horsepower criterion

From Equation (7.11), pressure drop across bit for the three given nozzle sizes (3 16s) is

$$P_{\text{bit}} = \frac{9.22 \times 10^{-5} Q^2 \rho}{A_T^2}$$

where

A_T = total area of the three nozzles

$$= 3 \times \frac{\pi}{4} \times \left(\frac{16}{32}\right)^2 = 0.589 \text{ in}^2$$

Hence,

$$P_{\text{bit}} = 231.179 \times 10^{-5} Q^2 \qquad (7.14)$$

Equation (7.14) can then be used to calculate the pressure drop across the bit for the given volume flow rates. Alternatively, an hydraulics slide rule may be used to calculate P_{bit}. Once P_{bit} is determined, P_c is then simply the difference between the standpipe pressure and P_{bit}. Table 7.7 summarises these results.

TABLE 7.6 Raw data for Example 7.2

Flow rate (gpm)	*Standpipe pressure (psi)*
300	500
400	850
500	1200
600	1700
650	1900

TABLE 7.7 Processed data of Example 7.2

Q *(gpm)*	*Standpipe pressure (psi)*	P_{bit} *(psi)*	P_c *(psi)*
300	500	208	292
400	850	370	480
500	1200	575	625
✓ 600	1700	830	870
650	1900	980	920

Figure 7.3 gives a plot of P_c against Q. The slope of this graph can be measured directly using a ruler or, more accurately, by a curve-fitting technique. The difference is usually small if the best straight line passing through the majority of points is drawn. Figure 7.3 gives a value of n equal to 1.47. The equations necessary for the calculations of the various hydraulics parameters are arranged conveniently as given below:

$$P_{\text{bit}} = \frac{n}{(n+1)} P_s \qquad (7.7)$$

surface pressure (2500 psi)

$$A_n = 0.0096 \sqrt{\frac{\rho}{P_{\text{bit}}}}\, Q \qquad (7.11)$$

$$d = 32 \sqrt{\frac{4A_n}{3\pi}} \qquad (7.12)$$

$$\text{IF} = \frac{Q}{58} \sqrt{\rho P_{\text{bit}}} \qquad (7.9)$$

where A_n = total flow area of nozzles (in^2); ρ = mud weight (ppg); d = nozzle size as a fraction of 32; and IF = impact force (lb).

For the current flow rate of 600 gpm Using Equations (7.7), (7.11), (7.12) and (7.9), we obtain

$P_{\text{bit}} = 1488$ psi

$A_n = 0.4404$ in^2

$d = 13.8$ or one 13 and two 14s

IF = 1179 lb

Optimum circulation pressure, $P_c = 2500 - 1488 = 1012$ psi.

Optimum flow rate

The value of optimum flow rate, Q_{opt}, is obtained from the intersection of the line $P_c = 1012$ psi and the P_c vs. Q graph. Figure 7.3 gives a Q_{opt} of 660 gpm.

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Using Equations (7.11), (7.12) and (7.9), we obtain

Q = 550 gpm

nozzle sizes = 14.3 or two 14s and one 15
impact force = 1295 lb

Maximum impact force criterion

From Figure 7.3, the slope of the graph is, again, 1.47.

$$P_{\text{bit}\,1} = \frac{n}{n+2} \times P_s$$

$$= \frac{1.47}{1.47+2} \times 2500$$

$$= 1059 \text{ psi}$$

Optimum circulation pressure, $P_c = 2500 - 1059 = 1441$ psi.

The intersection of $P_c = 1441$ and Figure 7.2 gives an optimum flow rate (Q_{opt}) of 840 gpm. Hence,

$$A_n = 0.0096\, Q \sqrt{\frac{\rho}{P_{\text{bit}\,1}}}$$

$$A_n = 0.7309 \text{ in}^2$$

$$d = 17.8 \text{ or two 18s and one 17}$$

$$\text{impact force (IF)} = \frac{Q}{58}\sqrt{\rho P_{\text{bit}\,1}} = 1390 \text{ lb}$$

Comparison

The results of Example 7.2 show that the BHHP criterion gives better hydraulics in terms of small nozzle sizes and higher jet velocities. The IF criterion gives a slightly higher impact force than does the BHHP criterion.

A practical check on the efficiency of the bit hydraulics programme

(1) Determine pressure drop across bit, P_{bit}.

(2) Determine bit hydraulic horsepower (BHHP):

$$\text{BHHP} = \frac{P_{\text{bit}} \times Q}{1714} \text{ hp}$$

or

$$\text{BHHP} = P_{\text{bit}} \times Q \text{ kW}$$

(3) Divide BHHP obtained above by area of bit to determine K, where

$$\boxed{K = \frac{\text{BHHP}}{\pi d^2/4}}$$

(4) For maximum cleaning, K should be between 3 and 6 HHP/in² (i.e. 3.74–6.94 Watts/mm²).

References

1. Brouse, M. (1982). Practical hydraulics: A key to efficient drilling. *World Oil*, Oct.
2. Robinson, L. (1982). Optimising bit hydraulics increases penetration rate. *World Oil*, July.

Problems

1. Using the maximum impact force criterion, prove that, for the case of limited surface pressure, the pressure drop across the bit, P_{bit}, is given by

$$P_{\text{bit}} = \left(\frac{n}{n+2}\right) \times P_s$$

where n = slope of circulation pressure vs. circulation rate; and P_s = standpipe pressure.

Hint: (a) Express the impact force, F, in terms of V, Q and ρ, to obtain

$$F = \rho Q V \qquad (IF = \frac{Q}{58}\sqrt{\rho P_{bit}}) \tag{P.1}$$

(b) Pressure loss across the bit is given by

$$P_{\text{bit}} = \frac{K \rho Q^2}{A^2} \tag{P.2}$$

where K is a constant.

(c) From Equations (P.1) and (P.2) obtain the relationship

$$F = K_1 Q P_{\text{bit}}^{0.5}$$

where K_1 is a constant.

(P-93)

(d) Using $P_s = P_{bit} + P_c$ and $P = K_1 Q P_{bit}^{0.5}$, and differentiating F with respect to Q, obtain the following relationship:

$$P_{bit} = \left(\frac{n}{n+2}\right) P_s$$

2. Using the Bingham plastic model, calculate for the well described below: (a) the circulating pressure, P_c; (b) the nozzle sizes; (c) the bottom hole pressure while circulating; and (d) the equivalent circulating density.

depth	= 9500 ft (2896 m)
hole diameter	= 8.5 in (215.9 mm)
drillpipe	= 5 in/4.276 in (127 mm/108.6 mm) 9000 ft (2743 m)
drill collars	= 8 in/3 in (203.2 mm/76.2 mm) 500 ft (152.4 m)
mud weight	= 13 ppg (1.56 kg/l)
yield point	$= 30 \dfrac{\text{lb}}{100\ \text{ft}^2}$
viscosity	= 20 cP
circulation rate	= 350 gpm (1325 l/min)
maximum operating pressure of mud pumps	= 2500 psi (172 bar)
surface equipment type	= 4

Answer: ((a) 2469, (b) three 11s, (c) 6480, (d) 13.12 ppg)

3. Repeat the above example using the power-law model.

4. Assume that the circulation pressure, P_c, is related to flow rate according to the equation $P_c \propto Q^{1.86}$. Determine the optimum circulating pressure, circulation rate, pressure drop across bit and nozzle sizes, using data from Example 7.2 and the BHHP and IF criteria.

5. Prior to changing the bit in a $12\frac{1}{4}$ in hole, the following standpipe pressures were recorded at different flow rates with the present bit on bottom:

Flow rate (gpm)	*(l/min)*	*Standpipe pressure (psi)*	*(bar)*
252	954	390	26.9
336	1272	620	42.7
420	1590	920	63.4
504	1908	1240	85.5
630	2385	1840	126.9

Present nozzle sizes	= three 16s
Present hole depth	= 6572 ft (2403 m)
Next hole depth	= 8300 ft (2530 m)
Mud weight	= 8.3 ppg (0.995 kg*l)

Flow rate not to exceed 600 gpm, to limit hole erosion

Determine the optimum nozzle sizes for the given flow rate. Also determine the optimum flow rate that the BHHP criterion yields.

Answer: Slope = 1.68; nozzles: one 13 and two 14s; $Q_{opt} = 660$ gpm.